ANSWERS
upon ſeveral heads in
Philoſophy,

FIRST
Drawn up for the private ſatisfaction of
ſome Friends; now expoſed to publick
view and examination.

By William Marſhall, Dr. *of Phyſick of the*
Colledge of Phyſicians, in London.

Dij bene fecerunt in opis me, quodque puſilli.
Finxerunt animi, raro & per pauca loquentis. Hor.

LONDON,
Printed by T. *L.* for *Nathaniel Brooke*, at the Angel in
Cornh[illegible] near the Royal Exchange and at
Greſham Colledge. 1670.

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The Firſt SECTION of PHILOSOPHICAL ANSWERS

CONTEINIHG

The firſt Six Anſwers upon ſeveral heads in Philoſophy.

By WILLIAM MARSHAL

LONDON,
Printed by *T. L.* for *Nathaniel Brooks*, at the Angel
in *Cornhil* near the Royal Exchange, 1670.

To the right Honorable *William*, Lord Viſcount *Bronker* Preſident, and the reſt of the learned members, of the Royal Society.

To the Incomparably learned Dr. *Francis Gliſſon* Preſident, and the reſt of the learned fellows of the, Colledge of Phyſicians in *London*.

To the worthily famous Profeſſors of *Gresham* Colledge in *London*.

The Author wiſheth all happines, ſubmitting theſe his Philoſophical anſwers to their grave cenſure.

THe things here [preſented, and which ſhall be hereafter preſented of the like nature, will vindicate mee] from both ſuſpition in you, and expectation in the Reader, of a dedicatory piece of flattery: I every where manifeſtly uſing a Philoſophical freedome.

15 Nov-43 Angew

[illegible] not ſo ill a ſervant either to [illegible] Honours, or to noble truths, which are the choice food of refined minds, to make this addreſs unto you for the protection of any thing which may deſerve a black cenſure. Let every line bear its own fate in its own boſom. Only being well aſſured, that theſe things in difference, and thus diſcuſſed between me and ſome friends; as they have been contrarily taught by perſons worthily had in ſingular veneration, both in the preſent and former ages; ſo they concern ſome heads of Philoſophy, which are of no ſmall conſequence unto true learning; and beſides, lying deeper in the pit of obſcurity then the firſt fadom, they ought to receive the ableſt deciſion; I knew not therefore how more candidly to ſtrip my ſelf of any concern pervicatiouſly to defend any thing in theſe papers, then by laying them and their fate at the feet of ſo noble and able Judges. Poſſibly the thanks of pri-

vate friends may have begot in me
ter opinion of ſome of theſe things
they may deſerve: your judgment may correct the errour of their indulgence. As all that deſire to learn, ought to wait upon you and your honourable ſocieties, as being together with the famous Univerſities and our Law Colledges, I mean our Innes of Court, as the ſoul and ſpirits of the Nation; ſo theſe Papers wait upon you, not preſuming in the leaſt to inform, but ſubmiſſely craving rather to learn of you what they ought to think of themſelves: according to the noble method of Philoſophy in ſome of your ſocieties lately inſtituted and purſued, not ſeeking for a precarious allowance, as a Rogue and Felone may find refuge and ſafety at an holy ſanctuary; but (however without the overweening dotage of ſelf opinion) either by their own fair truth and clearneſs to be juſtifyed, or elſe willingly to embrace the flames, which their er-

vours have deſerved. I know you will pardon the haſty rude dreſſe, or rather nakednes, (beſt ſuiting the purſuances of truth) in which theſe Anſwers to the queſtioning letters of ſome private friends, were firſt of all conceived: which without falſiifying could not afterwards be altered: and their higheſt ambition then was only private ſatisfaction, though now, except I will leave thoſe, for whoſe ſatisfaction they were firſt written, unſatisfied, they muſt proceed to put themſelves upon the teſt for publick, either juſtification, or diſallowance. The which I am the more eaſily perſwaded to, obſerving it a method allow'd and practiced both by antients and moderns to preſerve and deliver ſcattered points of Philoſophy by way of Dialogues and Epiſtles. And indeed this method hath this ſpecial advantage in it; we may in a word preſent what we have to ſay that is new, without loading the Reader with an unſavoury

cramb,

crambe, which hath been a thouſand times before in the preſs. As if theſe paters be able, which I leave to your ſerious cenſure, to juſtify themſelves: it is ſcarce poſſible for them to fall into the hands of any Reader, who ſhall not herein, if he read them diligently and with underſtanding, find, not only ſomewhat which is new, but as well that is memorable, and beyond all doubting, certain, and perhaps neither by himſelf formerly thought of, nor poſſibly every where in that vaſt Ocean of Books, which are in the world, to be met with. In theſe philoſophical oppoſitions, till I be called to account, I have for the honour I bear them, ſpared thoſe great and venerable names both moderns and antients, which I have herein oppoſed: it being my method in private, to purſue what is publickly in you exemplary *Viz.* to weigh controverſies and queſtions in philoſophy, not by parties and factions, time and conti-

nuanhuance, or number and opinion, but as near as poſſibly I can, by obſerving how the ſcales bear between the oppoſite reaſons: and I wiſh the method might be ſo happy, as to become more common. For the ſame reaſons I have ſuppreſſed the names of my ingenious friends, in oppoſition to ſome of whoſe thoughts theſe things were firſt of all written. Every day make your names more illuſtrious for clearneſs of judgment in the myſteries of nature. Being the deſire of all, and doubtleſs expectation of moſt and particularly of your Orator and ſervant

William Marſhal.

To

To his worthy and learned friends with whom theſe things were firſt debated; the Authors apology for the publication of theſe his papers.

TO ſome of you the name of an Apology might ſeem almoſt unintelligible, or little leſs then mockery: your intereſt being no leſs in the birth, then in the firſt conception of theſe philoſophical diſcourſes. Your quærying curioſity at firſt imprægnated the mind, with what your preſent importunity will not reſt without committing to the Preſs. To the reſt of you, my wiſh is, that with the favour of others, I might not have ſtood in need of this Apology. However let not the publication be miſ-conſtrued, as if I accounted all here debated to be peremptorily concluded, or all that I have oppoſed, conquered. The yet diſſenting of ſeveral of you from ſeveral of the things here by me own'd, might make a more hardy confidence then, I hope, I ſhall ever be guilty of, to become modeſtly jealous of its own judgment. If the things here made publick ſhould not be able to indure the piercing aire of this knowing age, however our free and mutual intercourſe upon ſuch and the like philoſophical doubts cannot want its juſtification. The publiſhing ſometimes of a miſtake is an occaſion of leading others into the right way. And becauſe every one cannot ſee croſs the Mediterranean, we are not therefore to walk either with hooded eyes, or to diſ-own the ſeeing of any thing at all, or what appearances things make unto us. Though the mind, which is the ſouls bed, be neither in all of the ſame ſize, nor in all equally luminous, or conceptive, yet each is equally concerned diligently according, to the proportion of his window, to take a view of the goodly ſcenes of nature. In theſe things ’t is better to erre then to be idle: as I am ſure you are equally perſwaded with

Your faithfull friend and ſervant
WILLIAM MARSHAL.

TO THE
READER.

THe publiſhing theſe papers hath now made thee my judge and cenſor: and they were no otherwiſe deſigned, but for every one freely to exerciſe his judgment about them. Moſt of theſe Problemes are more frequently upon the Anvile then they are found eaſily malleable into fair, clear, doubtleſs concluſions. Therefore if we miſſe the goale, it is but allowing one more in theſe ſublime things to run a planetary courſe. I ſhall in a word tell the worſt, which can juſtly be laid to their charge; ſome of the few things here diſcourſed, have left the commonly troden path; not as affecting novelty, againſt which upon other occaſions I appear for the an-

cients

tients, but as chuſing to follow reaſon rather then the name of any Authour. Firſt therefore examine their reaſons, and then give unbiaſſed judgment for the truth; which for the truths ſake I ſhall be ready to accept, though againſt me. For if thoſe to whom and againſt whom, though in a vayled way, theſe papers were firſt written, are ſuppoſed not to be exempted from errour, there is no room left but only for weak minds in theſe ſecrets of nature to arrogate unto themſelves an infallibility. If any complain of obſcurity, as both haſt, and brevity, and the ſubject matter, and the manner of writing, and the quality of the perſons for whom theſe things were firſt of all pen'd, may make them thereunto lyable enough, however I ſhall be ready to give both my account, and if valuable, my aſſiſtance to any who ſhall deſire it relating either to the obſcurity, or intricacy of any paſſage; and to accept from them any reaſons they ſhall be pleaſed to produce to the contrary,

Thine
W. M.

From my Study in
Nags head Court
in Gray's Church
Street.

A SYLLABE of the CONTENTS of theſe two SECTIONS.

In the first Section.

In the firſt Anſwer; is treated

Concerning the ſeveral Originals of Springs.

Concerning the irregular Ebbings and Flowings of diverſe particular Springs.

Of Chymical multiplication, or the encreaſing of the quantity of liquours by deſtillation.

Of the poſſibility of a perſous learning, in an hour or two, to write his mind in a language he is ignorant of: ſo as, what hee writeth in the unknown language, ſhall be conſtantly without errour in the Grammatical concerns of it, and the ſenſe good and continued.

In the ſecond Anſwer; is treated

Concerning ſympathyes between ſeveral parts of Animal bodyes.

Particularly, of the ſympathy which is between the breaſts and womb: and whether it be founded upon inoſculations of the Epigaſtrick and Mammary veins and arteryes.

Whether from view of the Breaſts, judgment may be made concerning the ſex of an unborn conception?

Of the Sympathetick relation of parts of the ſame ſide.

Of the forming of the Phaſes aud Shapes of the Moon from its mutuatitious light, and the proportion in which it hath its ſituation to the eye: and why the Moon appeareth to the eye rather plane then Sphæral. p.15.

In the Third Anſwer.

Freſh-water Springs are aſſerted at the bottom of the Sea.

Whether by frequent tranſhaping, and working upon Mercury, ſomewhat be not loſt irrecoverably, beyond all poſſibility and hope of reduction?

That the doctrine of the four Elements as uncompounded bodyes, cannot acquit it ſelf, againſt many rational doubtings and material controverſyes, to be a Doctrine unqueſtionable.

That the four Elements cannot be the firſt Elements.

Of the difficulty of bringing the firſt Elements to view.

That, in the Doctrine of the Ancients, ſeemeth a greater number of firſt Elements aſſerted, then four.

What may have been the true rational intendment of the Ancients, in their introducing the Doctrine of the four Elements. p.33

In the Fourth Anſwer ; is treated

Of the Sutures in the ſkull : and their diſtinctneſs according to their ſeveral kinds, number, and order.

How in the diſtinction of the ſutures, both as to their number and order, is a certain latitude, admitting variety in the method of their account, and the Anatomical Doctrine of them.

The like latitude is obſervable in ſeveral parts both of Concrete, and Abſtract Mathematicks.

Whether the præeminence of right-ſide parts above the left, be natural or arbitrary. p.45

In the Fifth anſwer ; is treated

Of the conſiſtency of immenſe volatility with immenſe ponderouſnes.

Of making tinctures by materation without addition.

Whether predictions may be concerning Comets before their appearance?

What is herein to be judged, of Comets which are by new amaſſement?

What is herein to be judged, of Comets which are *ſuppoſed to be permanent Bodyes, and not to be new amaſſements.* p.55.

In the Sixth Anſwer; is treated

Of the motion of the Celeſtial Bodyes about their particular Axes.

Whether ſuch their Circumaxall motion afford a convictive demonſtration and proof of the Copernican Syſteme?

That the Earths motion is not ſufficiently proved, upon the Hypotheſis granted, That Comets are neareſt the Earth, when they are in oppoſition unto the Sun. p.71.

In the Second

SECTION.

In the Seventh Anſwer, is treated

Concerning Angles of Contact.

In order thereunto, are given ſeveral definitions of Plane Angles, and diſtinct names to them, according to thoſe diverſityes of their kinds and conſtitutions, which, in this Queſtion, are more eſpecially material and neceſſary to be diſtinguiſhed.

The ſtate of the Queſtion is ſet forth.

It is ſhewn, that Recto-convexe Angles of Contact are truly Angles according to the definition of Plane Angles.

That Recto-Convexe Angles of Contact are neither deſtitute of Quantity, nor their ſides coincident.

In what ſeveral wayes, one Quantity may be greater then another.

Of Homogeneity.

what kind of Homogeneity it is, that is requiſite for proportionals?

That Angles have Figuration, as well as Quantity.

That in Angles is obſervable an Homogeneity or He-

Heterogeneity which is Mathematical, and alſo an Homogeneity or Heterogeneity which is not Mathematical, nor any way concerning their Quantity, but only their figuration, &c.

How three Angles being all Equal; the Equality between the firſt and ſecond, may be more abſolute then the Equality between the firſt and third being only reſpective.

How ſome Angles are conſtituted by the Concreſcency, Compoſition, and Annexion of ſeveral parts and Angles, which are Heterogeneal each to other.

VVhether like Angles, in unequal Circles and in like ſegments of Circles be alwayes equal;

That all Equal Angles are not like.

That all like Angles are not equal.

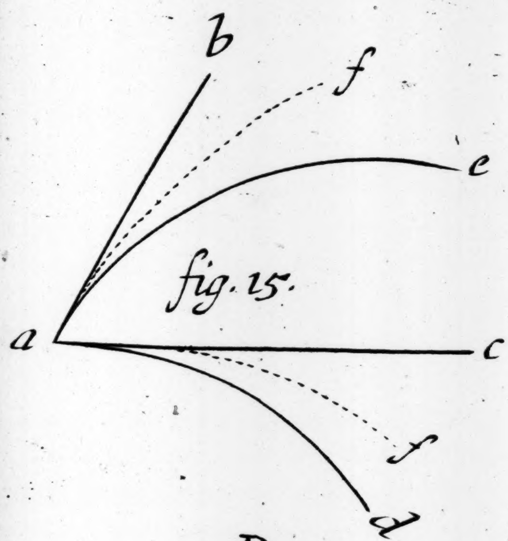
VVhether this Controverſy about the Recto-Convexe Angle of Contact, belong to Concrete or Abſtract Mathematicks?

VVhat was the firſt miſtake, that firſt miniſtred occaſion for the ſtarting of this controverſy, which hath been ſo long reteined, in theſe Learnings, in which nothing is more monſtrous and unuſual then pertinacious controverſy;

Theſe Eſcapes of the Preſs crave the help of the Readers Pen. *viz*,

PAg. 10. line. 11. read. aerial. p. 17. l. 2. r. heat. l. 16. 17. r. reaſon. p. 18. l. 10. r. eaſily. p. 14. l. 19. expunge the ſemicolon. p. 21. l. 28. r. Sympathy. p. 22. l. 15. 16. r. tumultuous. l. 24. r. thoſe. p. 23. l. 29. r. concerning. p. 24. l. 18. r. falls. p. 29. l. 7. r. confeſſedly. p. 33. l. penult. r. fluſhing. p. 35. l. 13. r. that that. l. 28. r. fire. p. 36. l. 13. r. us. p. 39. l. 1. expunge the colon: l. 16. r. fœtid. l. 26. r. their. l. 27 r. the. p. 41. l. 7. r. with the ſtring. p. 43. l. 12. r. to the violating. p. 44. l. 2. and 7. r. inanimates. p. 49. l. 3. r. ſeame p. 56. l. 9. r. certainly. p. 57. l. 15. for is regularly. r. irregularly. p. 61. l. 7. for if ſeeming. r. it ſeeming. l. 19. r. exact. p. 69. l. 5. r. limited. p. 71. l. 3. r. the Copernican. p. 75. l. 13. r. Jago. p. 80. l. 19. r. Lordſhips. p. 82. l. 17. r. like. p. 86. l. 5. r. BA. p. 89. l. 16. for mooving by. r. by mooving. p. 90. l. 2. r. an iſoclitical. l. 28. r. Be again. p. 92. l. 30. for AHH. r. AHF. p. 100. l. 6. r. even. l. 19. r. ſay. p. 101. l. 1. r. one or each. p. 124. l. 29. r. recto-concave. p. 155. l. 26. and 28. r. crooked-lined. p. 160. l. 18. r. not other. p. 187. l. 26. r. a mixt-lined ſecant angle or of p. 200. l. 26. r. crooked-lined. p. 232. l. 1. r. conſtant.

The faults of the Orthography are referred to the Readers ingenuity.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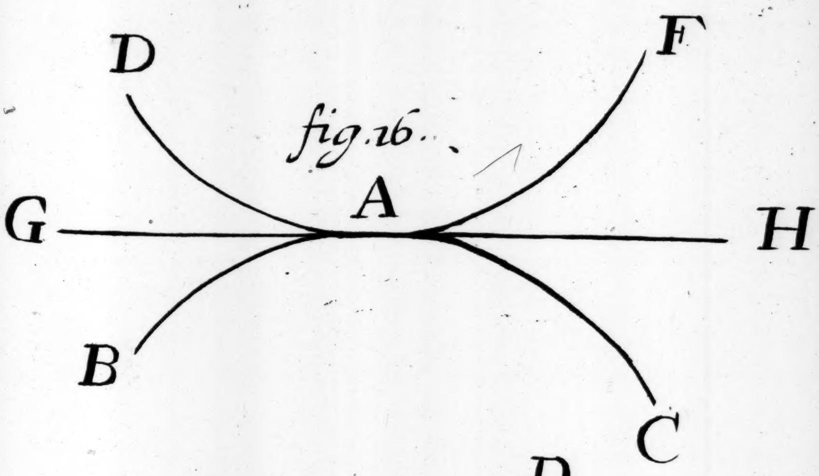
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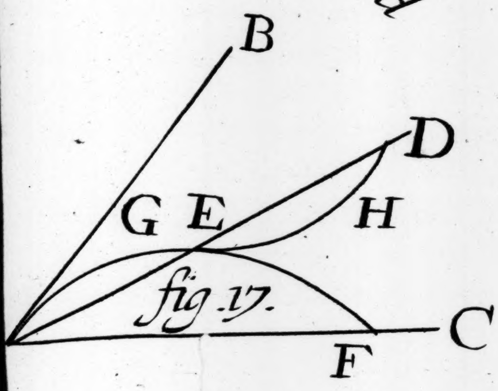
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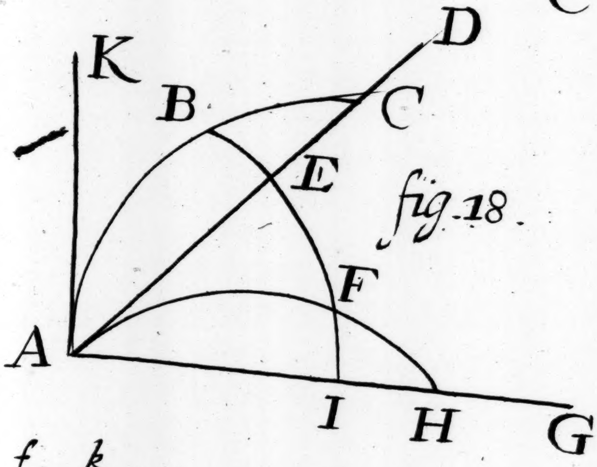
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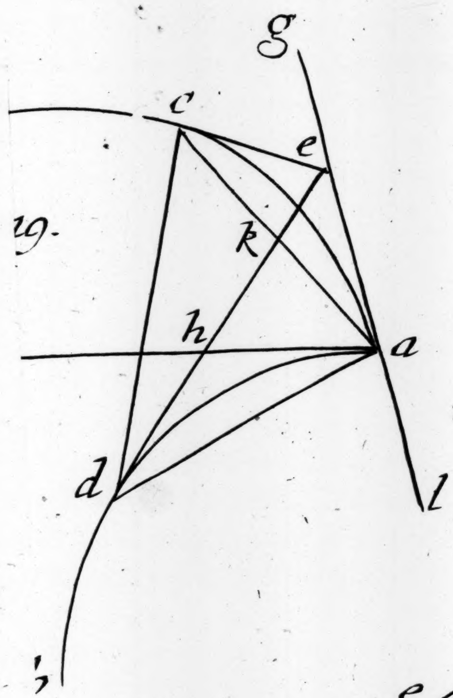
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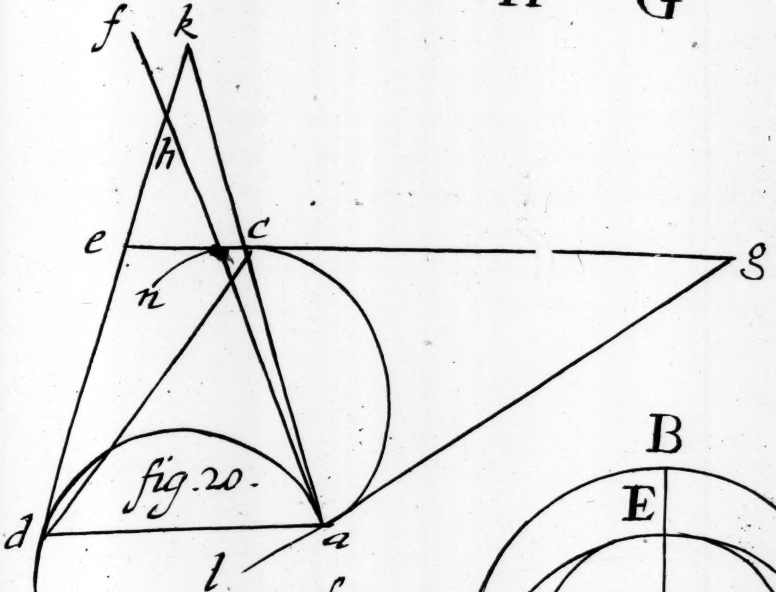
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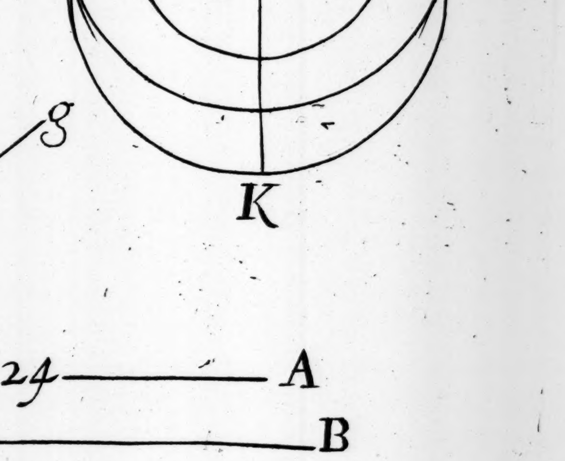
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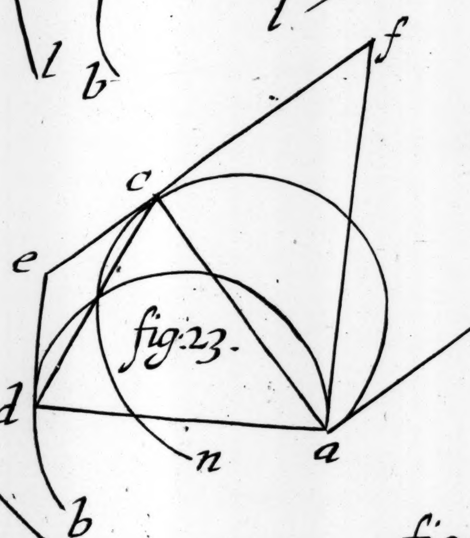
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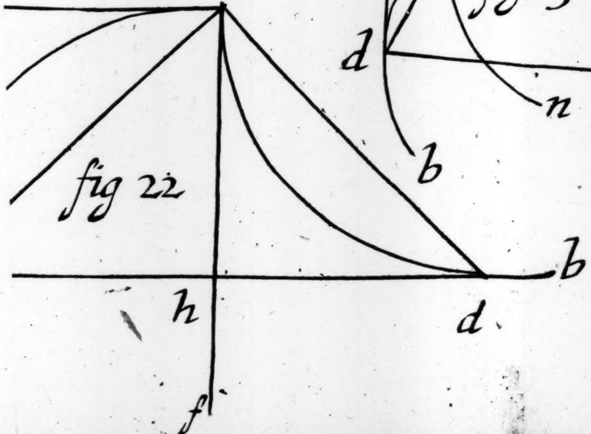
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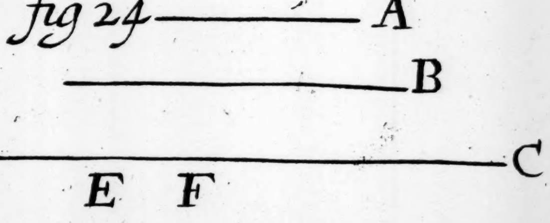
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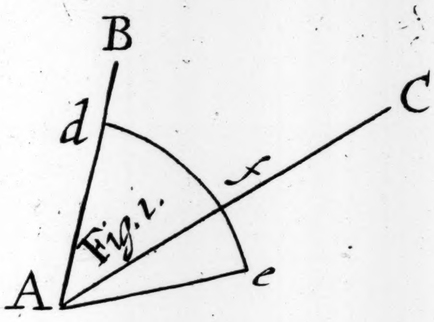
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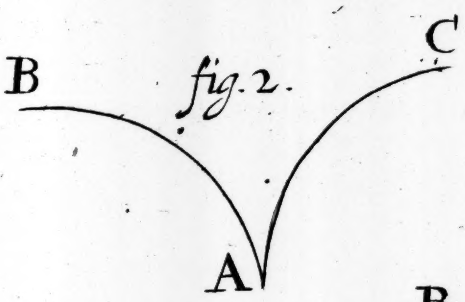
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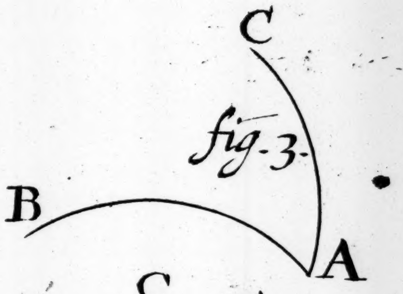
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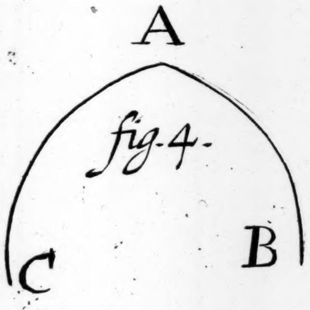
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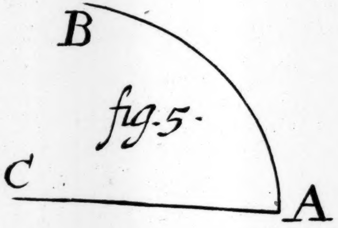
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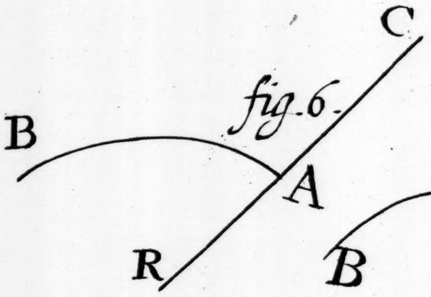
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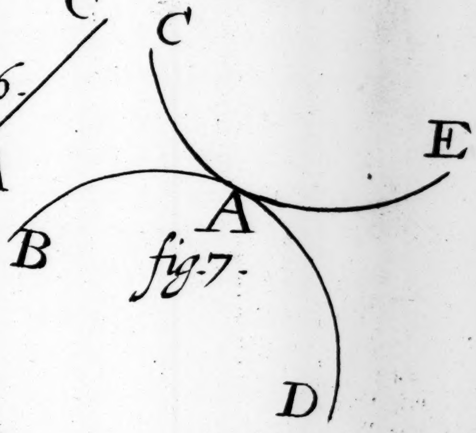
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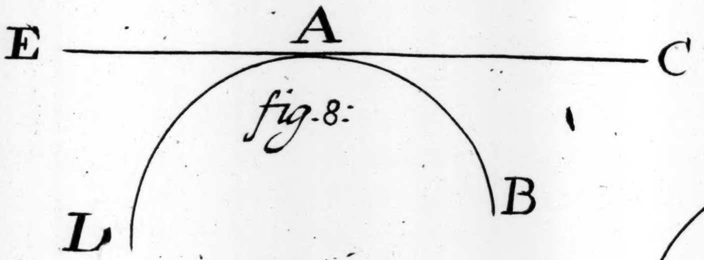
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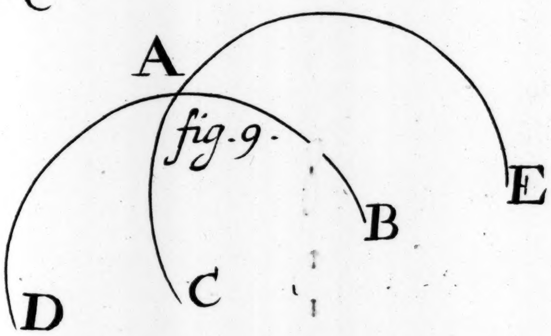
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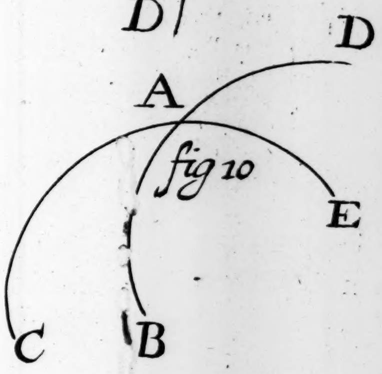
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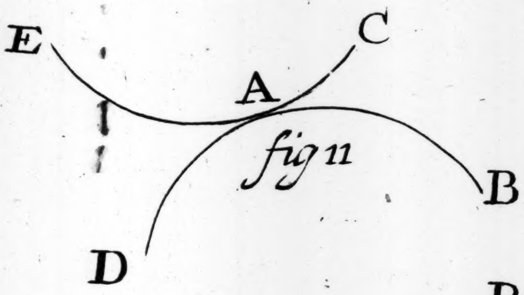
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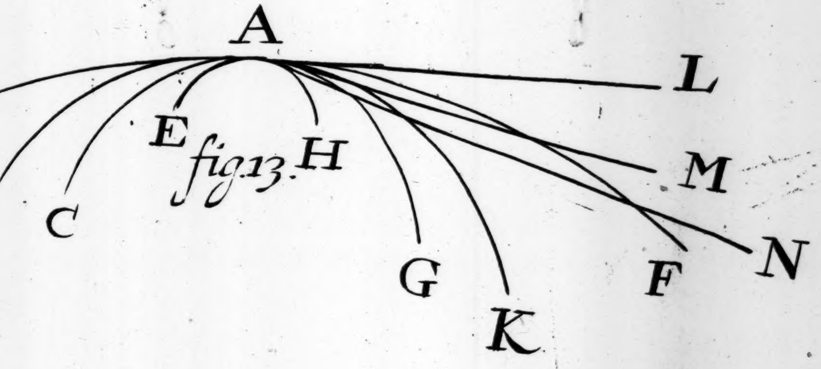
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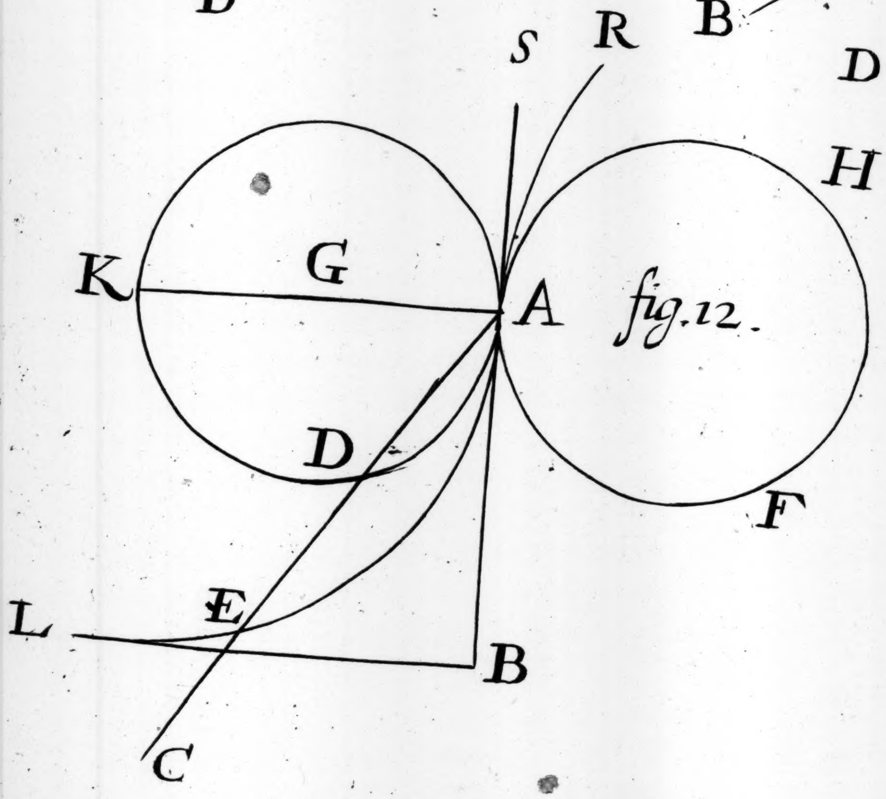
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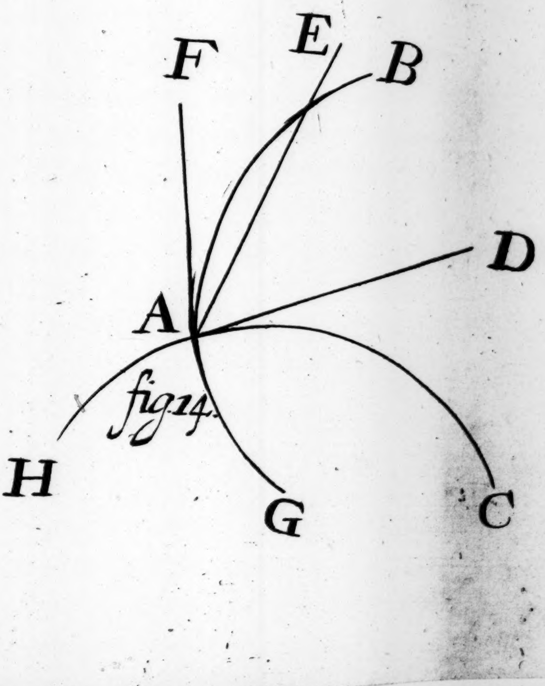
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ANSWERS
Upon ſeveral heads in PHILOSOPHY

The firſt Anſwer.

Concerning the original of Springs: that all Springs have not the ſame nor the like original: that all Springs are not from the Sea: that ſeveral things may by the way contribute as originals to the ſame Spring, whether by way of diſſolution, or condenſation. Alſo of the ebbings and flowings of particular Springs: in ſome more eſpecially obſerved to be in an unaccountable irregularity. Alſo of Chymical multiplication, or the diſtilling of liquors, ſo as by diſtillation to encreaſe their quantity to any given proportion. That Mercury is not explicable by the Doctrine of the four Elements. That in an hour or two's time a perſon of indifferent parts, able to read and write his native language only, may be taught to write his mind in a forein language, ſo as what he writeth ſhall be true in the forein language, and the ſenſe good and continued.

AS Nature in generation worketh the firſt beginnings of things uſually in a very dark loom, not permitting common eyes to have a view of its untaught art in ordering and warping its firſt filaments, covering with ſhells and membranes and divers veils the myſterious nature of firſt productions: The ſame judgement may be of Springs, which though every where obvious to the eye, by their pleaſant murmurings, cryſtalline pureneſs, and perpetual flux, delighting the mind, as well as the ſenſe; yet whence theſe Riſe, and how they are fed, and wherewith furniſhed, is more obſcure and ænigmatical than to be reſolved, as many eaſily imagine, in a word. And yet I ſuppoſe, an erroneous *Hypotheſis* may be the chief ground of difficulty in this Queſtion, while many generally expect that all Springs ſhould be of the ſame or like original. But why is that more reaſonable than to expect that all Springs ſhould ſend forth waters of the ſame taſte, colour, or virtue? Too many Inſtances, Examples and Experiments may be produced of Springs owing their original to the Sea, percolated thorow the Earth, to deny, or call in queſtion ſo evident a Truth, without a manifeſt crazing of our own judgement: yet that the Sea by ſuch percolations is not the original of all Springs, is as manifeſt, as true, and confirm'd by as many and weighty Experiences; not only of freſh Springs near the Sea, and in-land ſalt-ſprings; but eſpecially if we conſider the numerous Springs, which are in the tops, or ſides of ſeveral Mountains, vaſtly above the Seas level; and therefore not poſſible in any Channel to be naturally raiſed ſo high, without all ſtay, or im-

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the Eye of time grown clearer, ſeeth much more happily, as is by many ſuppoſed, without them. More craſs and thin, ſolid and fluid parts are not denyed, but apparently to be ſeen, offering themſelves to view in ſeveral plain and eaſie methods of analyſing Bodies; but the ſimplicity of thoſe parts, obtained in ſuch Analyſmes, is not yet demonſtrated. The ſtrange Monſter reported with ſo much averment to you, which ſeems ſo much to have ſtartled your patience, to wit, that a perſon of parts, educated only to read and write the Language of the Nation wherein he was born, may in an hours time be taught to ſet down any matter, ſecret, or other, in another language, as Latine, Greek, French, &c. and the Latine, or other Language to be true, and the ſenſe good and continued; believe me, though it be true, as it is, yet it is not to be eſteemed worthy your leaſt diſcompoſure: it may be done many ſeveral wayes, and yet when all is done, it is but a ſlight of the Brain; like many of the mimical Miracles, performed by nimble handed *Egyptians.* That there may be no room for diſtruſt of the truth of it, if affiance in me may ſo far prevail, you may be aſſured, I have ſeen the whole Model and Syſteme of it, in ſeveral Languages, beſides our own: and but that it would make my Letter of too great a Bulk, ſhould herewith have ſent you a Tranſcript of one of them. In this, as in the foregoing Probleme of multiplying Chymiſtry; in words ſpeaking and containing truth, ſomething ſeems to be offered to the underſtanding of the hearer, which is an *Herculian* ſhot beyond it. Pardon the length in which I have laboured diſcurſively to ſend back all the Quæries and Objections of your laſt fully anſwered.

ſhould the Patients Urine come in a few weeks to weigh more than all, both the drinks and meats taken by the Patient all along that time, and all the weight of the Body, whenſoever it was heavieſt during that time beſides? To contract all into few, the poſsibility of the Probleme, according to this explication, needs not be doubted, being an every where obvious work of Nature; as in the other ſenſe it ſeems vain and frivolous. And in reſearches about expedients for diſpatch; it is poſsible, here, as in other pieces of that ingenious Art, ſeveral Artiſts may have ſeveral methods and contrivances, to ſuck in ſtill more vapours in a continued ſucceſsion, and proportionably to condenſe them when ſucked in; alſo to diſpoſe the vapours, before they are ſucked in, for more expedite and eaſie condenſation at after: as every one has his peculiar Still, his peculiar Solvent, Bath, Ferment and Furnace. And that this point of Philoſophy were a little more cultivated were to be wiſhed, for the advantage of natural Philoſophy, and the concernes of it in ſome practical Arts: for ſo hereby the difference of the Airs at ſeveral ſeaſons, or at the ſame ſeaſon in ſeveral places, might come in ſome meaſure to be known, by the Taſte and Eye, as well as by the Noſtril: and the concerns of Rokes, Fogs, Miſts, and Airs any wayes altered, or infected, may be hereby more happily diſcovered in relation to the health and ſicklineſs of Man, and ſo of other Animals: ſo what difference there is between the Breathings of ſeveral Animals of the ſame , or ſeveral Sexes and Kinds, or of the ſame Animal at ſeveral times of its age, reſt, motion, ſear, joy, hope, love, anger, health, ſickneſs, peculiar conditions, or evacuations: and the diſcriminations between ſimple Airs, and the Airs altered by ſprinklings, ſtrewings, fumigations, from Minerals, Vegetables, Animals, may be made more evident. Your tying up *Mercury* to the old Elemental Laws, is a true ſemblance of the old Mezentian cruelty; and I ſhould pity the poor Fugitive, were I not aſcertained from his conſtant courſe, that as a ſlippery *Hocas*, he will never leave tranſhaping himſelf till, to your own confeſsion, he has ſlipt the Collar. Four qualities will aſſoon ſolve all queſtions, as four Elements; which ſome of the learned Antients, vainly hoped, not to ſay, fooliſhly boaſted, to do. As Philoſophers diſtinguiſh between the ſights of Age and Time; Age firſt ſees without, and after not without Spectacles: Time firſt ſees with them, and after better without them: the Elements were as Spectacles in the Beginnings of thoſe times, when men firſt ſet themſelves about the ſtudies of Arts, that helped them in a ſort to conceive ſomewhat of Bodies; but now niably apparent, that rains, in no contemptible proportion furniſh forth matter to be ordinarily by channelly veins convey'd, or elſe extraordinarily by a natural diſtillation wrought up into the Springs. And as this much diſcuſſed *Queſtion* is reſolved in theſe eaſie things, every where offering themſelves to view in Nature; ſo I doubt not, upon the ſame principles might alſo many other things, ſeeming at firſt ſight to be very myſterious, in the concern of Springs, as that which with ſo much admiration is by the rude ignorant people cryed up and obſerved in ſome, having, as they term it, their Ebbings and Flowings in ſuch irregular frequency as by no art can poſſibly be reduced to any certainty of account, or order: for the more ſpecial and clear obſerving of which they are wont to receive the water from the Spring into ſome ſtone, or ſuch like veſſel proportionately bored in, or near the bottom: all which in truth is without any retrogradation and reciprocation of motion in its channelly veins; being nothing elſe but an inæquality of the waters iſſuing from the Spring in equal times, either by reaſon of its more plenteous filling at ſome times the channelly veins of the Spring as it paſſeth, or elſe by reaſon it paſſeth with a greater or leſſer impetus; or poſſibly in ſome caſes and places upon both grounds: which what it hath in it that is more admirable than the running of any ordinary Brook, ſometimes with a fuller channel, ſometimes with a ſtronger ſtream, I do not yet underſtand: and how eaſily, upon the former principles, this is explicable any one may readily perceive: that we may more juſtly wonder how ſome grave Authors came to be ſo tranſported with the ſight or fame of ſome ſuch ſprings, occurring up and down in the World, as in their reports to offer them to the thoughts of diſtant Students, and ſucceeding times, as containing in them little leſs than miracle. And if the ſtudious would obſerve diligently what is in nature, poſſibly many other things, ſeemingly as intricate, might be capable of as eaſie explications; eſpecially conſidering the vaſt way a Spring may run under the Earth before it break forth; fairly inſinuated to us by the pure freſh water Springs, burſting up a great way within the floud marks of the Sea; clearly intimating to us, how two Hills at a conſiderable diſtance may both be concerned in the ſame Spring and its Courſe. And whereas we number and place Springs ac-according as we obſerve them to break forth and ſhew themſelves unto the day; in the true æſtimate, and upon laborious ſearch, it hath many times appeared quite otherwiſe; that the original of the Spring, has been at a great diſtance from the eruption, and the eruption has not been till after the confluction and meeting of, it may be three or four ſeveral Springs, every one carrying in it ſometimes the diſſolution, ſometimes the ſpirit, or ſomewhat of the firſt rudiments of ſome ſpecial Mineral; and altogether make up, not ſeldome, an almoſt inimitable compoſition. Your other Quæry concerning Chymical Multiplication, or the diſtilling of Water from a Pint to a Quart, if my Genius fail me not, when rightly propoſed and underſtood, cannot want ſome affinity with the matters we have already been treating of. For if any would impoſe that the ſame Water as by way of expanſion, without other additional water or matter, might by diſtillation be brought to a double, treble, and ſo a thouſand times as large dimenſions as before, as Wines, Vinegars, and other Liquors, have in them, without any further addition, ſo much Phlegme, and ſo much Spirit of ſuch a ſtrength, whether fixed, or volatile; it would require a very credulous judgement to allow the veracity, or poſsibility of ſuch a Probleme: and I am clearly free, to diſown the skill or power of any ſuch Chymiſtry; which once admitted, were it not for the ſhortneſs of humane life, and the hopeleſneſs of obtaining Veſſels, and Inſtruments large enough for the work, and conveniency to place them, the world might be in danger of Drowning from the lofty Artiſt; as once it was hypothetically threatned, with ſhaking by the noble and ingenious Mechaniſt. But I had rather underſtand a Probleme favourably, ſo as it may carry truth, poſsibility, and reaſon along with it: and ſo methinks may this be interpretable, to wit, ſubtilly to contrive, find out, and order expedients, ingenious helps, and advantages, to coudenſe free, and open exhalations into a watery form with diſpatch and expeditely, eſpecially at any ſeaſon, or time of the year: ſo as by this Art out of the arrial vapours within a limited time to give any limited meaſure of water, which reaſon can expect from Art, or without abjuring and putting off its own nature, handſomely make demand of. And that this is the Philoſophy of Nature appears in every days dewes, the trickling moiſture hanging on the chamber-ſide of the Glaſs, when the Air is very ſharp, and full of a nitrous coldneſs without: and we cannot rightly conceive the Idea of that famouſly known and frequently mentioned and in hot Countries, though rare, yet ſometimes by ſad experience atteſted diſeaſe, the Diabetes, without admitting both the poſsibility of ſuch a Chymiſtry, and of Expedients for the more ready diſpatch of it: for without an aptneſs to imbibe, attract and condenſe the vapid exhalations, which ſucceſsively come to be near and about the Patient, and that with very ſingular advantages for expedition, how poſsibly with ſubterraneous juyces, and moiſtures, cannot be denied by the obſervant, who almoſt every where under ground find leſſer veins of water diſperſed up and down the Body of the Earth, and in many places large concealed Rivers, elſewhere unfathomable Vaults, and Abyſſes: Sometime ſeen Rivers in their proper and perpetual Channels, make a dark courſe and diſcharge of their waters into the unſeen Bowels of the Earth. Upon the whole, in the method of this explication, waters are, in the ſame manner ſecretly within the Earth, raiſed to the top of the Mountains, in which, to our view, they are raiſed in the open Air to thoſe regions, from whence they fall back again upon us in the form of rain. And in ſuch vapid exhalations, it cannot be denied but in ſome places, and at ſome times, the Mountain ſprings ranging in a much higher level, may by poſsibility be furniſhed from the percolated ſea-water: but as the poſſibility of this is admitted, ſo the univerſal and abſolute neceſſity of it, may not be averred: it ſeeming hard to ſay, that the Mountain-ſprings what diſtance ſoever from the Sea, have no other way of being furniſhed, but only from thence. And, I ſuppoſe, it will as uneaſily obtain credit, that the Sea ſhould run as faſt every way, under the Earth, to the Originals of Springs, as the Springs generally do in ſeeking out their way unto the Sea. I doubt not,

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but beſides the Seas, there are under the earth many other liquors, contributing to the origination of Springs, ſome having their firſt riſe from diſſolution, others from condenſation, whence, and from the paſſages thorough which they run, and are percolated, riſe very often thoſe ſpecial vertues and dangers of ſome peculiar Springs: and ſometimes in a ſeeming præpoſterous way, though very conſentaneous to the true nature of things, Springs adjoyning to the Sea are freſh, and at a vaſt diſtance from the Sea, in in-land Countreys ſometimes ſalt. But beſides both the Seas, & thoſe other waters bred under the earth; its more than probable, the rains falling down in ſhowers from Heaven, adde not a little to the fluſhing and continuance of the Springs. So medical Springs after rains are noted for a while to be of leſs virtue; and in long droughts, 'tis uſual for very many Springs to be quite dried up, till rains fall again: and where, by long obſcure dens, caverns and paſſages under ground, acceſs may be had to ſubterraneous Rivers; ſuch Rivers, though lying vaſtly deep under the Earths ſurface, they are found to ſwell at after rains, and by their acceſſion unto the wonted ſtream, frequently do make a moſt hideous and horrid noiſe, full of terrour to thoſe in the Caves mouth, and at other times unuſual. From all which, ſeriouſly and impartially weighed, is made unde-

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pediment of percolation. Of which I have obſerved a ſpecial inſtance in a low, ſomewhat rocky, wedge-like Mountain, ſituate along the brim of the full Sea; but on the Land-ſide, at a little diſtance, ſet about with numerous and vaſt Mountains: in which little wedge-like Mountain, though moſt part immediately encompaſſed with a Plain, very conſiderably above the level of the higheſt, æquinoctial, or other Tides; was a perpetual Spring of moſt lympid, clear water, not unfamed for its ſingular medical vertues. It is ſcarce conſentaneous to ſenſe, reaſon and experience, to derive ſuch Fountains out of the Sea only by way of percolation; which though it be allowed to contribute, in ſome places, to the ſweetning of the fountainous flux, cannot be conceived apt to raiſe the water to an higher level than it had before. Not diſallowing the former therefore in its place; there is yet beſides it ſome other Original of Springs to be inquired after. And how perpetual Springs ſhould come to be in the tops of the higheſt Mountains of all, as the caſe of moſt difficult explication, cannot with greater reaſon and clearneſs be explain'd, than by bringing the waters thither, not in a watry form, but as vapours and exhalations, at leaſtwiſe the moſt conſtantly, and in the greateſt proportion; after the manner of a Diſtillation, ordered and managed by the inſtitutes of Nature; and this without Eſpouſing the Tenets of our new Platoniſts, that will have ſeveral perpetuous continued Orbes of Fires, diverſly graduated, for ſpecial Theological ends, contained ſtill between two and two perpetuous and continued Cruſts, or ſphæral ſhels of Earth, the one above the fire, the other beneath it, in reſpect of the Earths Center. And thoſe that know how under the ſurface of the Earth, whether plain, or mountainous, are frequent ſtores, both of Materials, and naturally formed Cavities, and other Inſtruments, eaſily accommodable to ſuch a work, will not conclude that to be a conjecture of fancy, which not only for its poſſibility, but the high degree of its probability, may juſtly claim to be entertain'd as a moſt real and undoubted phyſical Truth. For what doubt can be made of ſubterraneous heats and fires? and hot Baths and Springs, atteſt the, not only warmth, which is ſufficient for our purpoſe, but even the actual, and frequently intenſe, heat, and ebullition of liquors within the Caverns of the Earth, and the perpetuity, or conſtant continuance of ſuch ebullitions: and as the Earth is well known to be furniſhed with many natural Caverns, ſo in thoſe Caverns, for ſhape, proportion, ſituation, and other the like circumſtances, we cannot but imagine there is great diverſity. So how variouſly and plenteouſly the Earth is watered

with

The Second Anſwer.

Wherein of Sympathies between ſeveral parts of the ſame Animal Body in general and particularly of the Sympathy which is between the Breaſts and the Womb: and that the ground of that Sympathy is not founded upon the inoſculations of the Epigaſtrick and Mammary Veins and Arteries. That that Sympathy inſtructs not from the ſight of the Breaſt concerning the Sex of the unborn Conception. Yet parts of the ſame ſide more eſpecially Sympathize one with another, than thoſe that are on contrary ſides. That to found the Sympathy between the Breaſts and Womb, is no neceſſity of veſſels paſſing directly between the one part and the other: but the grounds of their Sympathy may be ſeveral other wayes clearly explicable, by manifeſt Arteries, Sinews, and other Veſſels. That the ſeveral ſhapes of the Moon have their original, forming, and alteration, from the parts of its mutuatitious light, and their ſituation in reſpect of our Eye. That there is not one general original of all light. And why the enlightned part of the Moon is ſtill toward the Sun, And why the Moon appears plane and not ſphæral.

I ſhould wrong the truth, if I ſhould not acknowledge, all your Arguments and Inſtances prove fairly the Sympathies, which nature has lodged in us, between ſome ſpecial parts above others; to which may be added by way of Declaration; that though no part in the whole animal Body, but is of affinity and concern unto all the reſt, yet ſome, beſides their general relation as of the ſame animal Body, have ſometimes not only one, but ſeveral other more ſpecial grounds of Sympathy, to tye & link them one to another; whether from homogeneity of ſubſtance, or commonneſs of original, or unity and concurrence in the ſame work, or concern in the ſame branchings of veſſels, whether Arteries, Sinews, or Veins, of what kind ſoever; each of which heightens the general relation which is between the parts: and the more of theſe are found inter-curring between any two parts, their relation is ſo much the more fortifyed. But of all other, the Sympathy which you urge, between the Breaſts and Womb, in animals is too univerſally known, and variouſly evident to be denyed, or doubted. How oft, among prying and ſuſpitious Women, has the ſtate of the Womb been diſcovered from the colour of the Nipples, aud condition of the Breaſts? And the judgements of Phyſicians juſtified and admired, when upon the manifeſt intercourſe between theſe two parts, eaſily transferring humours, whether natural, or præter-natural, from the one to the other, they have ſometimes predicted long before the diſeaſes of the Breaſts, or Womb? So upon expectation of the ones evacuation, the cure of the other has been ordered, and the event and time of recovery foreſeen? and obſerving how they grow and leſſen together, heal and fade together, become firm & flaccid together, repleniſh and empty together, except when the one drains the other, the intercourſe between them, and their common concern is manifeſt; however the grounds and paſſages wherein this intercourſe is founded and carried on, poſſibly may not be ſo very clear, as by many is preſumed. And though I honour you for honouring the Antients; it is now eſpecially leſs clear then ever; that the undeniable conſent and ſympathy, which is between theſe two parts, has its foundation in the inoſculations of the epigaſtrick veins and arteries with the mammeryes: Except we will ſo farr degrade the examinations of reaſons, and trials of experience, as to make what is antient, to be therefore indiſputably authentick; Concerning the ſympathy of the Breaſts and womb as held by the Antients) there are two things well deſerving ſerious & further conſideration: For hath not this ſympathy been rackt beyond its nature, and tentered beyond the truth, of what it is able to bear, or diſcover; when, upon the view of the Breaſts, declarations and predictions are made, not only of conceptions, & abortions in general, with a probable prognoſtick of the time and event of the feared abortion; but alſo from the difference of the Breaſts on each ſide, the ſexe of the conception, or abortion is peremptorily limited; and when twins of ſeveral ſexes have been conceived, the twin of whether ſexe ſhall alone be aborted is foreſhewn; in that rare & unuſuall caſe of aborting one twin, and compleating the due months of geſtation with the reſt? Not to alleadge more of theſe impertinent medical Enthuſiaſmes; ſuch groundleſſe and inconſequent deductions, as they are caſily received by the ſuperſtitious, who yawn after things novel and ſtrange, and blindly ſwallow what is offered without either præexamining by the eye of reaſon, or proving them by the præmaſtications of experience: ſo I doubt not but to ſober minds they will appear an apiſh overhugging the truth, to the ruine and deſtruction of it: an overchurning a ſerious, ſecret, true, obſervation of nature into ranckneſs and falſity; and a præpoſterous grafting a wild, wandering fancy upon the moſt cultivated ſtock of ſober, true and certain obſervation. For ſince men have ſet their minds to learn out of natures unwritten books, as well as out of the writings of Authours; what hath been more uſual than to find male and female conceptions indifferently on the one or other ſide, or horn of the womb? And though againſt experience no Argument ought to avail; we ſhall not ſtifle what they ſeem to have to ſay for themſelves;

That the left præparing vein riſeth from the Emulgent only, which may be ſuſpected, to be a weakning of its power and influxion; whereas the right præparing, or ſpermatick vein riſeth immediately from the great diſcending Trunk, of which the aforeſaid Emulgent is but an offſet. All this is generally and in moſt Bodies true: and in former Ages, with the gloſſineſs of its probability, carryed away the minds and judgments of moſt: but now upon more curious ſearches made into the Bowels, and heart of nature, the ſophiſtry, & weaknes of the former Argument manifeſtly appears: the ſpermatick veins, if any be ſo pleaſed to continue that name, being only reducing veſſels, and carrying forth from the vital fountain nothing at all to the Teſticles, womb, Ovaries, or other organes of generation; as may appear by obſerving; the motion of the juyces contain'd in them, notifyed from the ſide on which the tumour and fulnes ſuddenly upon an intercepting ligature gathereth. And the different riſe of theſe ſpermatick veins is more aptly referred to the indirect ſituation of the diſcending Trunk there; which uſually at the loins inclines a little rather to the right ſide, making the left Emulgent accordingly, to be commonly a little longer than the right: ſo as for a long tender venal veſſel to have had its original ſo obliquely remote, and ſloping over the fore bodies of the Rack bones, might not ſeem ſufficiently ſecure; however in the more ſtrongly coated arterial veſſels, it is notwithſtanding otherwiſe; the right and left ſpermatical arteries both riſing conſtantly from the deſcending branch of the Aorta. That Argument of the right ſide parts being, ſtronger, hotter, nimbler, &c. than thoſe on the left ſide; annexeth an incohærent ſequel, to a Truth which is meerly accidental: the ſpecial ſtrength of the right ſide parts not being any natural priviledge, but an advantage acquired by exerciſe, uſe and education; and by the ſame method, as eaſily transferable to the other ſide. And if at any time Twin parts be not equally concern'd, it is becauſe there is not equall relation to the ſides: ſo the Ovary of that Side to which the Conception is more principally affixed, in many females is of a fleſhy ſubſtance, of colour like Sanders, or ſomewhat brighter, the other all the while remaining without any alteration as a cluſter of watery Blebs & veſicles. But more particularly in the Copulations of mankind, all ſuch conſiderations of ſides muſt of neceſſity be obliterated; the right ſide of the one Parent being applyed to the left of the other, and ſo on the other ſide, the left to the right. In all this we deny not, but right ſide parts have more ſpecial relation to right ſide parts, where the parts twin, and are double: but to parts, which are ſingle, and poſited in the middle, Twin parts have equall relation. So as upon the whole matter, the diſeaſes of each particular Breaſt are more prone to be transferred to the ſame reſpective ſide of the womb, and thoſe of either ſide of the womb to the reſpective breaſt; of which are moſt apparent footſteps in the fabrick of nature: for though in ſome veſſels, as Blood-veins, water-veins, chyle-veins, arteries, &c. this diſtinction of ſides be again loſt and conſounded by inſertion into ſome common Trunk, before they can be traced from the one part, to the other collateral part; yet in ſome other veſſels without any ſuch confuſion in the midway, the ſame collateral veſſels ramify themſelves diſtinctly unto the parts all along on the ſame ſide: as is moſt manifeſt in the viſceral ſinew, or wandering pair: but yet hereby is not opened any window to ſee into the ſex of unborn conceptions. And, as herein I deſire to reverence antiquity, but follow reaſon; ſo upon your more ſerious thoughts, I doubt not but you will judge, with me, another Point upon this ſubject very well worthy calling in Queſtion: and that is the much noyſed ground of this ſympatly between the Breaſts and womb; ſaid, and formerly thought, to be the inoſculations of the mammary veins and Arteries

with

with the Epigaſtriks. It is well known, how many lateral heterogeneal inoſculations, between Arteries and Blood-veins, have been introduced by cloſet Anatomiſts; while, in Bloudleſs ſpeculations, they formed nature according to their reaſonings, to ſolve appearing phænomena's; inſtead of forming their reaſonings according to what they ought firſt to have obſerved in nature: the verity and authority of all which, among Anatomiſts at ſharp, that carry their Eyes in their hands, and will believe no more than they ſee, is at preſent not much; however formerly ſolemnly, and for ſingular purpoſes, placed in and very near ſeveral of the chief Bowels: yet the ſumultous æſtuation which this Hypotheſis continually placeth nature under, like the meeting of two contrary ſeas, may be ſufficient, in reaſon alſo, to decry and diſcard it: for which reaſon alſo lateral homogeneal inoſculations of bloud-veins with bloud-veins, as they are frequently found, ſo they may as eaſily in reaſon be admitted. But the inoſculations in the Queſtion being final; I wave theſe that are lateral. And final inoſculations are on all hands, agreed to be in a manner impoſſible to be ſhewn; becauſe, when granted to be, yet there where they are, the veſſels are ſo minute, and ſlender, that it neither can be eaſy for the quickeſt ſight, nor the moſt cautions hand, either, without hurting them, to come at the veſſels, where they are inoſculated, or to diſtinguiſh the veſſels when heterogeneal at their inoſculation, or to diſcern a final inoſculation, whether of veſſels of the ſame, or of different kinds. So that this whole Queſtion about final inoſculations being to receive its deciſion by the judgment of reaſon; ſuch final inoſculations as are homogeneal, as of Arteries with Arteries, and Bloud-veins with Bloud-veins of the ſame Trunk, ſuch as in this Point were underſtood and intended by the Antients, are no way capable of defence, allowance, or juſtification at the Barr of reaſon: for it experimentally, and to the eye, appears, that if any ſuch inoſculations ſhould be ſuppoſed, the ſame Humours at the ſame time, from the ſame place, ſhould move two contrary ways, to wit, both to, and from the heart, and in veſſels riſing from, or gathered at after into, the ſame trunk: in which account, the lung vein, the hollow vein, and the port vein, though all of them bloud-veins, yet becauſe of the diverſity of their Trunks, are to be eſteemed as heterogeneal. And finall homogeneal inoſculations generally, and therefore alſo between the Mammaryes and Epigaſtricks, being thus everted; the old doctrine oncerning the ground of the ſympathy between the Breaſts and womb cannot be retained. And if any think by a ſhort alteration of, and putting heterogeneal inoſculations for, the rejected homogeneal, the ground of this ſympathy will remain firm and clear as of old; to wit by making humours to be transferred from the womb by the Epigaſtrick arteries and mammary veins to the Breaſts, and by the mammary arteries, and Epigaſtrick veins from the breaſts to the womb; I ſuppoſe, that though ſuch inoſculations be not as the other, impoſſible, though undemonſtrable to ſenſe, yet upon mature conſiderations, theſe will not be judged ſufficient to found this ſympathy upon. If we lay the matter ſeriouſly in the ballance, even ſuch heterogeneal and final inoſculations do rather hinder the transferring of humours from the one part to the other: to be ſure, look how much falls, or is any ways drawn into their capacities and cavities, cannot by them be ſo transferred, but is otherways diſpoſed of: for the mammarie veins carry not to the breaſts, but to the ſubclavians, and ſo to the heart, and the mammary arteries carry not to the breaſts, but to the Muſcles and parts on the forebelly. In like manner the womb will appear equally unconcerned in thoſe veins and arteries: for the Epigaſtrick veins carry not bloud unto the womb but unto their collateral Iliacks, and ſo to the heart: and the Epigaſtrick arteries carry not bloud from the Iliacks to the womb; but to the Muſcles lying on the fore part of the belly. And of themſelves, neither mammary bloud-veſſels reach the womb, nor either of the Epigaſtricks the breaſts: ſo as, notwithſtanding any thing in theſe veſſels, we yet ſeem to be in the dark as to the grounds of this confeſſed ſympathy. Not to add, that in many Animals ſome, and, in ſome all, the breaſts are quite out of the way of the Mammaryes, on the ſame ſide of the Epigaſtricks with the womb, and, on that ſide, as farr removed from the capillar terminations of the Epigaſtricks and mammaries, as the womb it ſelf. And profeſſedly I underſtand not, upon what neceſſity the veſſels founding, or contributing to the ſympathy of theſe two parts, muſt needs paſs directly between the one part and the other; and may not as well here, as in the ſympathy of other parts be admitted ſufficient for this purpoſe, by their common concern and relation to ſome ſpecial branch, or general trunk, And veſſells thus related, touching and terminating in theſe two parts, are not hard to be found in ſeveral kinds; as arteries and ſinews to convey like matter to both theſe parts: veins and ſinews to reduce what is improper, ſuperfluous, uſeleſs, or redundant in either, or meet and apt to be transferred from the one to the other: in which both cannot but be aſſiſted by the agreement of their ſimilar attractions, and that conformity of ſubſtance, which is either conſtant, or at ſome ſpecial times between ſome, or all of each of their parts: which as it may be a ground of the maturation & exalting of theſe parts together, ſo thereby the Bodies, pores, and paſſages, of both, are alike open for the reception of like humours: and the ſame conform ſubſtance may be the cauſe why thoſe vital emanations & irradiations which the generative parts receive from the parts which are principal; when, I ſay, thoſe vital emanations come to be remitted and reflected back again from the generative parts to all the reſt of the body; then, above all others, the breaſts gather into themſelves powerfully thoſe forces of nature, which are ſo reflected back again from the generative parts univerſally to the parts of the whole. And ſuch a courſe, as I have hinted, is both plain by view, undeniably certain, and familiar in the uſual method of nature; the breaſts, above other parts, being exonerated by the evacuations of the womb and the evacuations of the womb, more eſpecially then of other parts, being diverted by the excretions of the breaſts: ſo that theſe being the common and trodden paths of nature, there cannot but remain an open way alſo for the tranſlation of other matters. And though by the Mammaryes and Epigaſtricks, whether inoſculated, or not inoſculated, I have above ſhewn, that ſuch conveyances cannot be made; yet nothing hinders, nay 't is very apparent, that ſuch tranſlations may be aptly enongh made, nay no doubt are made, by the thoracick and preparing veſſels; to wit, by the Arteries and ſome ſinews directly, as bringing humours directly unto the parts, and by other ſinews and veins more indirectly, as they are apt to carry, or not carry away what has by other veſſels been transferred from one of theſe parts unto the other. I had diſcourſed theſe things ſhorter, if I had not been concerned to quit my ſelf of raſhneſs, with which you had attainted me for ſeeming herein to have made a defection from the Antients. I ſhould aſsoon defend the Moons Eclipſses to be a celeſtiall fainting, and planetary ſickneſs, and to be cured by Cymbals, Drums, Cornets and Trumpets; becauſe the Antients thought ſo. Yet for the ſame truths ſake I account my ſelf bound to juſtify the old Aſtronomers in the cauſes by them aſſigned of the ſeveral ſhapes of the Moon, as depending upon its mutuatious light, the proportion in which it is received, and the poſitions from which it offers its ſelf to our view; all concerning the Earth in the ſhapes of its lucid parts, ſave in its Eclipſses ſometimes when it is at the full, being the product merely of ignorance and want of obſervation. I am not of opinion with many, that there is but only one fountain of light, which is the ſun; to which by imaginary refractions, and reflections, and ſtrange labyrinthæan windings they would referre all other lights; ſending the moſt glorious of the celeſtial Bodies a borrowing to this fountain; while evidently in theſe inferiour Regions, in the mineral, vegetable and animal Kingdoms, &c. not to ſpeak of the cæleſtiall flames, ſeveral ſpecial fountains of light have been, and are frequently obſerved, though neither ſo full, nor durable, and perpetual as thoſe above. However we cannot but confeſſe the Sun to be the Fountain of the principal Lunar light; as another more faint light diſcernable in all parts of its body at whatſoever age, ſo it be out of the Combuſtion of the Beams of the Sun, which lye betwixt us and the Sun, or the Beams in vicinity to them, cannot be denyed to be innate unto it. And the roundneſs of the Moons and Suns bodies, being evident to the Eye in every poſiture and revolution, and apparent in Solar and Lunar Eclipſes, beſides ſeveral other demonſtrations of their rotundity; it follows from hence by neceſſary conſequence that the Moons body cannot be guilded all over, at the ſame time, with ſuch luminous beams as it borrows from the Sun: ſeeing in any two ſphæres, in whatſoever poſition, the one without the other, there is ſtill a certain part of the ſurface of the one, unto which without penetrating its body from no point of the other can be drawn a ſtraight line, as is eaſy to demonſtrate. And the magnitude of the bodies of theſe two Stars, being confuſedly unæqual the one unto the other, and the Suns the larger; it follows in like manner by the ſame neceſſity, that though all the Moon cannot be at once enlightened by the borrow'd Solar light, yet at all times, more than half of the Moons ſphæral ſurface, is ſo enlightened; except in thoſe caſes when the interpoſition of the earth, or other opake bodies, obſtructs the ſolar radiations from falling upon it. For if a leſser, and a greater ſphære be put and adapted into an Iſoſcelary Cone, whoſe Axe is perpendicular to the circular Baſe at its Center, the Lines in which the ſurface of the Cone toucheth the ſurfaces of the ſphæres, ſhall be two Circles leſs than the reſpective greateſt Circles, each of its own Sphærs as is not hard to demonſtrate. And it is as apparent, that the Iſorrhopical center of that part of the Moons ſurface, which is ſo ſolarly enlightened, is ſtill the point directly obverted to the Sun; thorow which point a ſtraight line drawn between the centers of theſe two Stars paſseth. So as the enlightened part of the Moon being ſtill towards

wards the Sun, no wonder, if in their conjunction the Moon cannot be ſeen at all; as chiefly becauſe of its combuſtion, ſo alſo becauſe the enlightened ſide of it, is turned away from us towards the Sun: that though, by deſcending into deep pits, Stars above the Horizon may be ſeen at Noon day, yet the Moon in this caſe cannot by ſuch helps be in like manner ſeen, its dark ſide being towards us, and the moſt that our Eye can at any time at one view ſee of the Moons ſurface, being ever leſs then the half of it, as is demonſtrable. Upon the ſame grounds, at the Oppoſition, i. e. full Moon, the enlightened part of the Moon as it is towards the Sun, ſo it is towards us alſo, from whence follows, that all, that we can then ſee of the Moon, is enlightened, and that much more of it is enlightened, than it can be poſſible for us to ſee. In the Quadratures, whether of increaſe, or decreaſe, the ſtate and poſiture of the enlightened, and unenlightened parts of the Moon, in reſpect of the Sun and us, offer themſelves in a leſs entire, divided, middle manner: to wit, the parts of the Moons ſurface obverted unto us are but, as it were, half obverted to the Sun, and the parts of the Moons ſurface obverted to the Sun are but, as it were, half obverted to us: ſo as what is obverted to us in the Quadratures is, as it were, half enlightened, and half unenlightened and at other not Cardinal times, the nearer that the Moon is to the full, ſo much the greater part of the Moons ſurface obverted to us, is by the Sun enlightened: and the nearer the Moon is to its Change, whether paſt, or to come, ſo much the leſser part doth the Sun enlighten of the Moons ſurface obverted to us. And that theſe things are really ſo, may be from hence confirmed: at whatſoever Age of the Moons creſcency, or decreſcency, if you obſerve its luminous parts ſeen by us, however horned, or bunched, the Sun is ſtill on the luminous ſide of the Moon. Now the cauſe, why the Moons luminous parts appear plain and flat, rather than bearing out, like a Sphæral ſurface, or a portion of a Sphæral ſurface, is becauſe the inequality of the viſual beams, between the Eye and the ſeveral luminous points, is not ſo proportionate as to be diſcernable by our ordinary ſight: for without ſuch inequality, to be diſcerned in the viſual beams, neither ſenſe, nor reaſon can conceive true Idea's of a ſpheral ſurface. And upon theſe Hypotheſes and principles all phænomena's concerning the Moons ſhapes, as to magnitude, proportion, continuance, ſituation, alteration, alternation, being perpetually and accurately ſolved; the Novelty which would dethrone this doctrine ſo apparent, ſo rational and unconvincible of the Antients, as it is unworthy the acceptance of the conſiderate and judicious, ſo it cannot but be unmeet for them to impoſe and obtrude upon others; to omit the groſſe inconſiſtency involved in it, which I ſhall in my next at large unfold, if theſe things fall ſhort of giving Satisfaction.

The third Anſwer.

That there are freſh waterſprings at the bottoms of the moſt antient ſeas. Whether Mercury *by frequent tranſhaping it ſelf, and often reduction looſe not ſomwhat of its powers and virtues? That the Doctrine and being of the four Elements, as unmixed bodies by their mixture making up all other bodies, is not unqueſtionable. In the four Elements may be allowed to be the general and moſt common lodges of the Worlds firſt Elements; but themſelves at the moſt can but be allowed to be only ſecondary Elements. How nature may ſo ballance the firſt or ſecond Elements by ſome ſpecial Symbolical properties among them, as to elude all the endeavours which art can poſſibly make for the bringing of the firſt Elements to view and light. That the number of the four Elements and of their properties or qualities ſingly or in Conjugations aſcribed to them ſeemeth inſufficient to veſt them in the right of firſt Elements; upon the knowledge of whoſe natures, all Phyſical phenomena ſhould be capable of explication. That the antients ſeem to have allowed a greater number of firſt Elements. A conjecture what the antients might rationally at firſt deſign at their firſt introducing the Doctrine of the four Elements.*

SIR.

THat freſh water ſprings lye at the bottom of the ſeas, both frequent, and with flaſhing iſſues; is to me upon ſeveral obſervations, not undiligently made, as abſolutely certain, as to you it ſeems impoſſible: and this I judge not only in the ſeas, which have made inchroachments by inundation upon the antient bounders of lands, which before were plenteouſly up and down watered with ſprings; but as well, there being the like reaſon, in thoſe ſeas, which are able to plead the higheſt and moſt antient præſcription, and cannot be any other wayes chargeable, then in their dayly fluxes and refluxes, with the leaſt new invading of the earths boſome, and of thoſe wells of coldboiling-natural nectar, with which it is uſually there ſtored. And why ſhould natures opening a vein of freſhwater into the ſea, ſeem ſuch a ſea monſter when at land we ordinarily meet with divers ſprings, of different virtues and originals, meeting at after together in the ſame channel; from their concourſe and mixture conceiving ſecret virtues, manifeſt alterations, and ſpecial properties, as ſtrange to the illiterate and unexperienced, and generally wondered at by the moſt, as the boiling heat appearing preſently in the ſuddenly mixt Oyles of Tartar and Vitriol, though cold, when poured together. I ſhall not urge, that ſome, would have the ſeas, proportionately to their depth, freſh at the bottom. I only move, if this be not more eaſy, to be aſſented to, then what is dayly ſeen, and therefore not to be queſtioned in point of truth, that the ſea fiſhes &c. Though continually living in that briny pickle remain ſtill however freſh: many times the ſea-fowle, that moſtwhat fly but about it, ſometimes ſwim in the ſurface of it, ſenting and taſting much ſtronger of the ſea, than the fiſh that live deep and conſtant in it. But Experience being the grand Umpireſs in the Queſtion, in aſſurance of its convictiveneſs, I forbear at preſent. For your reducing of *Mercury* after all operations of fire, whether actual, or potential upon it, I have no reaſon to cheriſh ſuſpitions of the truth of ſuch performances: but that reduction ſhall be ſo perfect, as to give back the *Mercury* as abſolute in all virtues, as it was, or could at firſt be delivered, will not be eaſily conſented to, by thoſe that know that fire burns the chief wing upon which in Amalgames *Mercury* carryes along with it Gold: and the force of fire upon it, is in the nature of a rape robbing it of that virgin treaſure, to which the noble mettal is ſo ſequacious; which once loſt, can never be reſtored again to the deſloured *Mercury*: as in all volatiles that which is of nimbleſt wing flyes firſt, and the higheſt ſpirit firſt: and when an impregnation is to be made by fire the work of Philoſophy is judged chiefly to lye in the governing, of the firing, that, like the Sun, it may give enlivening heat, and not become, as a deſtroying Element. The reaſon, why ſo

few

few are acquainted with the excellency and prærogative of this pure *Mercury*, is becauſe generally it is a fire-burnt *Mercury*, which is at firſt delivered to us. I omit that in a thouſand inſtances, after Art has ſeparated the natural union of parts, though it may again unite them, yet that re-union will in many points fall ſhort of the firſt natural union. But why does it ſeem ſo monſtrous to you, to call in Queſtion the exiſtence of the four Elements, it being a doctrine, which in all Ages has been attended with doubts? if you think you can ſhew as *A*ir, and Earth and water, yet what ſhew can you make of Elemental fire? And ſince the diſſolution of the Orbes, there being no Concave of the Moon, what region is deſigned and intended for it? I preſume both Hearths, and Altars, are too low, to be the proper Sphære of this high Element. And upon the whole, as little can be ſaid for any of the other three as for this: the State of the *Q*ueſtion and controverſie being rightly underſtood. For it is not fire that is denied, or Earth, or Ayr, or water, but all as Elemental. How great a ſhare and proportion theſe have in making up this part of the worlds ſyſteme, is too evident to ſenſe to be called in Queſtion: many leagues of Earth, or ſea, anſwering to each ſingle degree: but that theſe four are the firſt uncompounded compounding bodies, of which all others by their mixtion conſiſt, and are made; ſeems, from hence, to follow by a very ſickly conſequence. Not without ſolid conſideration, and ſufficient cauſe, did the Antients honour the Earth with the ſtyle of the All-feeding-Earth: what innumerable vegetables and Animals ſpring, live and grow in its boſome, not to touch at the ineſtimable treaſures lodged in its bowels? Others, and of the worlds ſages, with honours not unequal, aſſign unto the water not only Beauty's birth, but the Riſe and original of all things: and every moment convinceth the uſe and neceſſity of Air &c. Theſe are Arguments of their univerſal uſe, concern and excellency, but do not prove their Elementalneſs, no more then that one Tree is the Element of another, becauſe the one is engrafted into the other, or that the mother is the Element of the child, while it lives annexed unto her in the womb. That in theſe are the general and moſt common Lodges of the worlds Elements, from which each nature may furniſh it ſelf with what is convenient for its being, nutrition and growth, may eaſily be conſented unto: but that theſe are the very Elements unmixed and uncompounded, will be an Herculean task to make forth by any Argument, or experience, ſo long as each in its region appears repleniſhed with all variety of Beings. What a numerous diverſity of Earths are to be found in the Earth, of diverſe colours, weights, virtues; ſome healing, ſome ſcouring, ſome binding, ſome Alexipharmical, beſides chalkes, and Marles, and ſeveral clayes, ſands and gravels, noble quarries, rich mines, coals, bitumes, marcaſites, ſalts, minerals and metals? ſo in water, all coagulable vapours and exhalations meet condenſed with the alluvion and diſſolution of various ſalts and other minerals, together with manifold ſubterraneous Oyles and ſpirits; whence the wonderful diſference of their weights, ſents, colours, taſts, conſiſtence and operations. In like manner the Air, what is it but an uncertain, unconſtant, randome compoſition of all ſorts of fumes and vapours; according to the nature and poſition of the Atomes diſperſed in it ſometimes clear, ſometimes cloudy, ſometimes healthful, ſometimes peſtilent, ſometimes delighting, at other times offending the ſenſes? And as difficult will it be to find and ſhew any uncompounded Fire. But you will ſay, that though theſe be not Elemental Earth, Fire, Aire and Water, it hinders not but there may be ſuch Beings and bodies; I anſwer, I have no abhorrency againſt the opinion, if ſo by any practice, or in any diſſolution theſe Elements could but be ſhewn, and their ſufficiency to explain the phænomena of nature: but thoſe parts, which, generally

nerally in diſſolutions: are offered to be accepted for theſe Elements, appear nothing leſſe, and, beyond all diſpute, very diſtant from uncompounded natures; and the analogous parts in ſeveral diſſolutions, as different one from another, as one of the Elements can be from another Element; which is repugnant to the nature of firſt Elements; as is uſually and truly urged againſt ſalts, ſulphures and *Mercuries* to put by their claim in firſt Elementarineſſe, in the mixtion of bodies: *Viz.* that in ſeveral diſſolutions, the analogous parts anſwer not in the leaſt one to another, except it be in ſome very general, and external conformity; but one is ſowr, another ſweet, another ſharp, another fælid, another fragrant, thus expanding into all manner of variety, wherein ſhould be nothing but the pure ſimplicity of a firſt Element. In a good ſenſe, and with fair Explications, I have been ever ready to acknowledge their intereſt as ſecondary Elements, as a doctrine conſentaneons to the Sacred Traditions of Theology; or as before, as the principal Lodges of the firſt Elements: but neither that which fils the regions, nor that which remains after their diſſolution of mixed bodies, could ever yet to me ſeem capable of the denomination of firſt Elements. And poſſibly it may be one of natures myſteries never to per-

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mit us to ſee the firſt Elements naked ſo curiouſly ballancing them in their connexions, that all attempts of Art, for their diſcovery, ſhall be eluded: ſo any Artifice to reſolve and anatomize a natural body into its firſt principles, grounded upon the fixedneſs and volatility of the parts, is eluded, if firſt, or ſecond Elements of the ſame fixednes, or volatility be connexed: ſo if grounded upon diſſolubleneſs in a certain Menſtrue, it is eluded by connexing ſeveral firſt, or ſecond Elements æqually diſſoluble in that, or the like Menſtrue: ſo corroſious by ſeparatories, fumigating, anointing, cementing are all eluded as to the bolting forth of the firſt Elements, if they meet with ſeveral firſt, or other Elements æqually paſſible in thoſe operations. And the very number of the Elements has not ſeldom rendered them unto me very ſuſpitious, that they could not be firſt Elements. For by what fair Argument ſhal the Quaternion of them be demonſtrated? if, as is uſual, we argue from qualities, whether ſingular, or in ſymboliſmes, I ſee not how this number can be maintained: more qualities, and more conjugations of them, ought by the ſame reaſon to prove a far greater number of Elements And alas, how vainly light do they render themſelves, that, by four conjugations of diſputable qualities, ſeek to ſolve all the phænomena which are in the world ; of the Loadſtones pointing northward , its drawing untoucht ſteel , the Load-touched ſteels drawing of other ſteel , the bleeding of the ſlain at the preſence of the Murtherer , the moving of an untouched Lute-ſtring being Uniſon , or in a ſtrong conſonancy ſtring that is moved , the ſinking of perſons drowned the firſt dayes , and then floating to the top at after , they have layd ſoaking ſo long in the water , the Cramp fiſhes aſtoniſhing the Fiſhers hand at the long diſtance of line and Angle , the ſtrange Ebbings and flowings of particular ſeas and ſprings , and a thouſand other? Upon the old principles , how lame is the doctrine of ſolidneſs and fluidneſs , of opacity and tranſparency , in bodies ſometimes of the ſame ſolidnes , ſometimes of the ſame fluidnes , beſides a million of other inſtances , whoſe reſolution mateth the doctrine of the four Elements? However miſ-underſtand me not , as if I denied Elements , or firſt Elements : theſe muſt either be allowed , or no mixtion ; of which we have lucide examples in every corner of the world : for it was a Golden-rule in the ſchool , which now I am oppoſing , that the ſame , or like , as ſo , working upon the ſame , or like , as ſo , cannot make any alteration : that I ſuppoſe I have good reaſon not to be forward to embrace the new received opinion, ſo much cheriſhed by ſome perſons of Eminency and parts, that there is only one Element of all things licked into ſeveral external ſhapes and forms. But this controverſy not concerning your judgment, it were trouble now to purſue an impertinent. The reſt I ſuppoſe, any where defenſible, ſave at Stagira; where Fire muſt burn, Air poyſon, Earth bury, and water drown, whatſoever ſhall be ſuſpected in, or alleadged againſt the Traditions of the imperial Philoſopher. However my thoughts are ſtill comported to yield up themſelves to that doctrine upon the firſt clear and experimental demonſtration of it; if any ſuch might be hoped. In the interim I want not fair footſteppings in the Antients to induce me to think, that when they make Bodies to be compoſed of that which is hot, and that which is cold, of that which is dry and of that which is moyſt; they mean nothing leſs then theſe four bodies, or rather vaſt Amaſſements, vulgarly now known by the name of the four Elements: but they rather intend, the true Elements of nature, it may be ſeldom, or never ſeen alone; in which, ſuch qualities, or properties are ſpecially eminent: and they are ſo far from reſtraining natural compoſitions to the at preſent received *Quaternary* of Elements, that conſtantly at the ſame breath they hint many other:

adjoyning in their recitals of natural mixtions, temperaments, and compoſitions, to that which it hot, cold, moyſt, dry, Elements endowed with ſeveral other properties, as that which is ſalt, that which is ſowr, that which is bitter, that which is ſharp, that which is ſweet &c. and the extremities and diſtempers intolerable unto nature, whether in general the Animal nature, or more eſpecially the Humane nature, are made to be, when ſuch Elements ſever from their mixture, and go apart by themſelves to violating, and breaking up that ſweet proportionate compoſition and harmony, which is ſo friendly and grateful unto nature. As the Pyrotechniſts analyſe bodies, upon the conſideration of ſeveral differences and contrarieties which are in their compoſitive parts, as that ſome will abide the fire, others not; ſome be wrought upon by ſuch diſſolvents, or corroders, others not: ſo the Antients to reduce all inanimate things to four heads, may have inſiſted on ſeveral contrarieties, paving the way to ſuch a doctrine: *Viz.* that ſuch bodies are either firmly ſolid, or elſe fluid: and thoſe that are fluid, are ſuch, either in a groſſe corpulent matter, or in fine exhalations: and thoſe exhalations, either apt to conceive a flame, and burn, or elſe of a nature, thereunto indiſpoſed. And according to ſuch a diſtribution, many various bodies fall aptly enough under the ſame head of Amaſsment: and all inanimals in the world may be eaſily reducible to ſome one of theſe four heads. But then theſe four heads are not particular firſt Elements, uncompounded, and ſo conſtitutive of things in mixtion; but general notions, unto which the inanimals of this lower part of the univerſe, are aptly reducible. Favour theſe Conjectures, which obtrude nothing, only labour what they can to juſtify the methods of the Antients. Send back my inſtruments and the laſt tranſmitted Problemes with your firſt convenience.

The Fourth Anſwer.

Concerning the ſeams of the Skull, and the number of the ſeams by which the Skullbones are joined one unto another, over and beſides the three principal proper and true ſeams. That without contradiction, variety may be admitted both in the number and order of ſuch ſeams. That neither abſtruct nor concrete Mathematicks are wholly exempt from a liablenes to ſuch varieties. That the pre-eminence of right-ſide parts is no way natural in ſuch as twin but only at pleaſure.

SIR.

THe learned men were riper for controverſy, than became the gravity of their profeſſion, or elſe upon the Point, which you have tranſmitted to me, held by each with ſo little real, though a manifeſt nominal difference, it had not been eaſy to have occaſioned the hot language contained in your report. Though the one make but five pairs of ſeams, by which the Temple bones are joyned to the wedge bone, and alſo to the two Crown bones, and thoſe (beſides the three principal ſeams) to the forehead bone, or bones, and to the Nowl bone; and the other, contrary to the general doctrine of the Anatomick School, aſſerts that there are ſix; and both make their appeal to Autopſy and the Skeleton;

I ſee not wherein either is diſabled to maintain his Aſſertion, as both agreable to truth and juſtifiable upon the view: only not without ſome rational ground for the difference, *Viz.* either hath, as I ſuppoſe, his different method, upon which he inſiſts in making up the account of the ſeams. For the number will differ according to the ground by which you will diſtinguiſh and make out the unity of each ſeam: as alſo according as the ſeams are accounted ſingly, or by pairs; from which laſt manner of accounting may ariſe alſo a third different number of them, though ſtill without any change in the thing and object. Not being privy to the terms upon which this controverſy was between them managed; this Reconciliation, though unqueſtionably true, and pertinent to the Queſtion, as by you in general propoſed, may however poſſibly be impertinent to it, as by them ſtated and defended: though I underſtand not upon what other terms the Aſſertions of both can be capable of juſtification: and I am not willing to think otherwiſe, between perſons, ſo eminently learned. It cannot be denied but in theſe nicer things obſerved in the head, and diſtinguiſhed uſually by numbers, as the ſeams, and holes of the ſcull, there has not wanted ſome ſeeming diverſity and obſcurity: while it has been thought, that merely at the pleaſure of Authors and of thoſe that ſit in the Anatomical chair, ſometimes one thing, at other times another, and not conſtantly the ſame thing, is made to give the order, unity, integrity and diſtinction of ſeams. So the Nowlebones connexion with the Crown bone, and Temple bone of the ſame ſide, is generally conſtrued to be two ſeams, and of two diverſe kinds; the one true, the other mendous, or defective: though ſo fair be the continuance of the one ſeam unto the other, that the one is uſually known by the name of an Additament unto the other: and yet in other ſeams diverſity of bones doth not, with all, make multiplicity of ſeams; as in the Lepidoidal, or ſcaleſeam where the Templebone is joyned to the Crownbone and to the wedge-bone by a ſuture, generally accounted but for one: ſo the connexion of the inner proceſs of the forehead bone, to four ſeveral bones of the upper chapp, is by moſt accounted but for one common ſeam. The like, or worſe perplexities occurre ſometimes in the account of the holes in the head; while ſome of the holes, which are common to ſeveral bones are accounted in each bone: others, in like manner common only in one of the bones, whence many times the number of the holes pertinent to each bone is left confuſed, and imperfect. For the reverence I bear unto the great Artiſts, in whom theſe things may be ordinatily obſerved; I am not willing to impute them any otherwiſe then to their great intentneſſe upon matter and things; by which they ſeem overborn into a certain negligence of names and appellations. And thoſe whoſe ſtudy is rather to be Anatomiſts, then ſo eſteemed, and draw their Inſtitutes and obſervations from the Body, more then out of Books, will eaſily conſent with me that in this whole Doctrine, of the ſeams, and holes of the head, names and numbers are of leſs moment, ſo there be a clear underſtanding of the things themſelves. So none, who has truly made uſe of his own Eyes can deny ſome parts, or ſides of the true ſeams of the skull to be mendous, and ſome parts, or ſides of ſome of the mendous to be in a ſort true: that in ſeveral of them the diſtinction of true from mendous, and ſutures from harmonyes, ſeems to be oft more matter of name impoſed at pleaſure, without clear and ſufficient ground for real and peremptory diſtinction. Things thus ſtanding, in the conteſt you mention; how eaſy is it to make, on the ſame ſide, the ſeams, by which the Temple bone and wedge bone are joyned one to another, and (as before) to the Nowl bone, Crown bone and Forehead bone, to be at pleaſure, either five in number, or if you pleaſe ſix; For the one, or the other number follows, according as you ſhall take the connexion of the Temple-bone with a Crown-bone and the wedge-bone, to be one ſeams as it is moſt uſually accounted; or elſe to be two; for which, being the connexions of two and two ſeveral bones, there wants not, as appears, ſome fair and probable ground. And thus though I would not charge with falſity, that whoſe truth is from real grounds ſo plainly defenſible, howbeit ſomewhat varying from the ordinary methods, and ſyſtems of Authors; yet I judge it not meet we be too forward, by reaſon of our more clear inſight into things, in altering, ſlighting and rejecting the Terms, names, diſtinctions and methods, which have, time out of mind, been in continual uſe with learned men. For names, that were firſt coyned for diſtinction and inſtruction, if they ſhould be dayly and hourly changed, would be ſure to introduce confuſion. And though in Names and numbers the Antients have uſed ſomewhat of authority, they have notwithſtanding, in the ſame ſubject, fairly exerciſed their reaſon. So, with a very little allowance, this whole doctrine of the ſutures, has been caſt by them into a Mould methodical enough: the ſum whereof in few may be this. The ſutures of the head either concern not, or elſe concern the bones of the upper chap: thoſe which concern not the Bones of the upper chap, are

either

either largely toothed ſutures, *Viz.* the Crown ſuture, the Dart ſuture, and the Lambda ſuture; which may alſo receive another ground of diſtinction from their concerning the two Crown-bones connexion one with the other, or with the Forehead bones and the Noul-bone: or otherwiſe; other ſutures, not concerning the bones of the upper chap, are not ſo fairly not deeply, ſometimes ſcarce at all, at leaſt not ſo manifeſtly dented, or not in the ſame manner; as the Lepidoidal connexting the Temple-bone to the Crown-bone and to the wedge-bone; the connexion between the Crown-bone and wedge-bone; the connexion between the Noul-bone and Temple-bone, commonly called the Additament of the Lambdoidal; the connexion between the Nouls additament and the wedge-bone; and the connexion between the Fore-head bone and the wedge-bone: which may alſo receive another ground of diſtinction from their concerning the connexion of the Temple-bone aud wedge-bone one with the other and with the Noul-bone, Crown-bones, and Forehead-bone. So the ſutures, which concern the bones of the upper chap, either concern ſome one, or more of them in common with ſome of the aforementioned bones; or elſe they concern, the bones of the upper chap alone, as connexed mutually amongſt themſelves. The ſutures which concern ſome one, or more of the bones of the upper chap, in common with ſome of the aforementioned, make connexion only with ſome of theſe, *Viz.* the Temple-bone, wedge-bone, or Fore-head-bone; for the Crown-bones and Noul-bone are no where connexed to the bones of the upper chap: And they are particularly theſe; *Viz.* the connexion of the outer proceſſe of the Fore-head bone, with the inner proceſſe of the firſt bone of the upper chap: the connexion of the firſt bone of the upper chap with the wedge bone: the connexion of the inner proceſſe of the Forehead-bone with the fifth, fourth, ſecond and third bones of the upper chap, being in the order recited, connexed from within outwards: the connexion of the outer, or yoke proceſſe of the firſt bone of the upper chap, with the yoke proceſſe of the Temple bone, in the middle of the yoke bone: which yoke bone is not a diſtinct and ſeveral bone, but made up of two proceſſes of two ſeveral bones: and the laſt is the connexion between part of the upper, part of the diſsepiment and the wedge-bone. Laſtly the ſeams which concern the bones of the upper chap only among themſelves are the connexion of the firſt and fourth bone, the connexion of the ſecond bone with the third and fourth, the connexion of the third and fourth bones, the connexion of the fourth and fifth bones, the connexion between the two fifth bones, the connexion of the fourth and ſixth bones, the connexion between the two ſixth bones, the connexion of the other part of the upper part of the diſſepiment with the fifth bones, the connexion of the lower part, of the diſſepiment in the Plough-ſhare bone with the ſixth, *i. e.* the palat bones and ſome ſmall part of the fourth bones. In all which, if ſomewhat at pleaſure, they have uſed authority in the numeral order of the bones, and in the account and order of the ſeams, it is no more than is freely allowed to each Artiſt in his Art, ſo long as the Authority they uſe, neither bears, nor creats any repugnancy to the ſubject matter. And ſo even in abſtract Mathematicks, ſomethings are determined, not by neceſſity, becauſe they can be no otherwiſe, but by long uſage and authority, becauſe they may conveniently be as is determined. So Arithmetical numerations, which are every where received and taught in Decuple proportion; with as much facility and Mathematical accurateneſs might be inſtituted in undecuple proportion; and then the uſual Probes, ſhould not be by caſting away of nines but of tens; or in ſexage cuple proportion; and then the Probes ſhould be by caſting away of fifty nines: or non cuple proportion, and then the Probes ſhould be by caſting away of eights. And the like inſtances might be given in Geometry; and in ſeveral, if not moſt parts of Concrete Mathematicks. So in ordering the Account of the Brain ventricles, of the ſides of the ſame bones, and ſometimes of the branchings of ſinews, veins, Arteries, other veſſels, and the like; custome, and authority, commonly gives præcedency to the parts, cavities, or branchings, poſited on the right ſide; though without offence unto Truth, all Accounts might as exquiſitely be compleated in a quite contrary order. But though theſe things might be, and as well, as what is; yet what is, being as accurate, as what otherwiſe might be, it is ſufficient to know that theſe things might be otherwiſe methodized, without renouncing the Antients method. I have, you ſee, too high an Opinion of the perſons concerned, to imagine their difference could ariſe from, ſome peculiarities in unuſual skulls; in which it is not uncommon to find great difference in the numbers of bones, ſeams and holes, not only under diverſity of Age and ſex, which, is ſo common, it is not otherwiſe to be expected; but as well when no ſuch cauſes can be alleadged, for the diverſity*: nay, many times in the main and principal, commonly called True ſeams we do not ever find a conſtant regularity and uniformity. But of thoſe things, which

ſeldome

ſeldome are, we are not to maintain Controverſy, as if they were the way of nature. This, Sir, I have written not to Inform, but confirm your judgement; which I know ſo well verſed in the Syntaxe of this our humane body, that it cannot diſſent from what is.

The Fifth Anſwer.

That immenſe volatility may conſiſt with immenſe ponderouſneſs. That tincturs may be altered by maturation without any addition, Whether the appearing and motion of comets may be before their appearance predicted. That no ſuch particular predictions can be made concerning the meteors which are in the Atmoſphear of the earth, nor of the firſt appearances of ſuch Comets as are ſuppoſed to have their Original from new amaſſements of Cometical matter in other Atmoſpheares; though after their firſt appearance, upon ſome obſervations accurately made, ſomewhat, though nothing ſo peremptorily as in other caſes, may be predicted, relating to their future motion. That it is not impoſſible, but concerning Comets which are permanent bodies, and not new amaſſements, predictions may be made long before, of their future appearances and motion.

SIR.

I Take your ſatisfaction, upon my laſt propoſed, clear, and doubtleſs Experiments, now ſince by you proved and approved, as a fair acknowledgment of that Truth, which however to me upon its former evidences needed no farther confirmation: *Viz.* that ſo different are the genuine notions and qualities of ſixednes and gravity, that immenſe volatility may and is ordinarily conſiſtent with an im-

immenſe ſpecifick ponderouſneſs; ariſing not from the moles and quantity of the bodies under conſideration, but rather from their natures and kinds. That what in your firſt velitation you aſſumed as abſurdly groſſe, inconſiſtent, and impoſſible; is now, upon your own acknowledgments, moſt eaſily and obviouſly demonſtrable by experience. No leſs certainly is to be acknowledged, in what you propoſe for the tingeing of metalline bodies, only by ordering and attending them in the management of their maturations, without the addition either of body, or ſpirit, as being all the time of this operation under the ſure ſeal. Could I perſwade my ſelf it ſhould not be reſented as a grand unkindnes, to be ſilent in what you call for my thoughts in, in the cloſe of yours; *Viz.* Whether prognoſticks at certainty, ſuch as are of Eclipſes, of Coitions, oppoſitions, motions of other Starrs, whether, I ſay, ſuch prognoſticks and of like certainty, may not alſo be made of Comets, of their Appearances, common impediments removed, and of their motions; I ſhould, if it might be, herein, lay *Harpocrates* finger upon my lip, and ſeal them up into a pertinacious ſilence. Not but I am deſirous to know, and willing to ſearch after truth: only I fear me, theſe are ſecrets of nature, by their peculiar myſteriouſnes, ſinking themſelves ſo low into the pit of obſcurity, that the ſtock of obſervations and diſquiſitions about them, which the world as yet hath, is not able to raiſe them ſo high, and place them ſo near day, as to be within humane reach and diſcovery. What I now offer is a Cæſarean birth of the mind, not brought forth by me, but cut out of the womb by your importunity: if it want ſhape, licking and lineaments, accept it as an unripe Abort, and either hatch it to perfection in your Thigh, or give it a little duſt to cover it. I take by way of præſumption, that in this Quæry and Queſtion you mean not by Comets, any of thoſe more uſual, leſs permanent is regularly moved Meteors, bred, gathered, fired, and burning in the Atmoſphære of our Earth; after their Appearance, there cannot be much certainty of their Motion, though ſometimes there may be conjectures probable enough, and in the Event by obſervation juſtifyed, when the fuel, or ſovent matter of ſuch Meteors, is manifeſtly, upon what account ſoever, known to be only, or moſt copiouſly ſituate and diſpoſed ſome one particular way: but of the generation and firſt appearance of ſuch Meteors, particularly the moment when, and the point, or exact place where, they ſhall appear, much leſs of certainty, in ſuch pronoſticks is to be expected; there being ſo much variety, contingency, and uncertainty, in the cauſes and meeting together of thoſe cauſes, which contribute to their production. And though there are, and have been many prædictions in general, and Rules of prædicting Aſtrologically, or Phyſiologically, from the ſeaſons of the year, the temperature, and diſtemper of ſeaſons &c. Concerning ſuch meteors in general; yet in a matter ſo unallyed unto certainty, repleniſhed with all manner of caſualties, to promote, or retard ſuch productions, I have not known any offer peremptory Pronoſticks of the kind of the meteor, its ſhape, magnitude, duration, motion, with abſolute determinations of its time and places, at any time beforehand. The quality and condition of the ſubject matter, making it as impoſſible to bolt out ſcientifical and oracularly certain predictions of ſuch meteors, as it is, a year before, to prove, or ſhew that in ſuch an hour, and in ſuch a quarter, ſhall be a Rainbow, ſo colour'd, ſo continued, or diſcontinued and of ſuch limited dimenſions: or that, ſuch a moment, from ſuch a point of ſuch an Azymuth, ſhall a devolant Star ſpring forth, which, in its fall, ſhall run obliquely thorow ſuch and ſuch Azymuths, and expire at ſuch an height above the Horiſon; or that ſuch an hour, in ſuch a Longitude and Latitude, ſhall in ſuch Altitudet and poſitions, and of ſuch dimenſions, be ſeen by day four Suns , or by night three Moons. To lay aſide therefore the conſideration of theſe, as ſuppoſed impertinent to our preſent purpoſe ; what may judiciouſly be concluded upon your *Q*ueſtions concerning thoſe other Comets, which lye without the compaſſe of this Earths Atmoſpære? If there be not two ſorts of ſuch celeſtial Comets ; there are at leaſt two ſeveral and very different Hypotheſes and notions under which they are conſidered by Artiſts: and Artiſts of equal worth and fame, order their reaſonings, ſome upon the one ſuppoſition, ſome upon the other : *Viz.* Some, as if Comets were new made bodies, amaſsed, and gathered in ſome of the ſuperior Atmoſphæres ; many of which are, not without fair reaſon, ſuppoſed to be in thoſe vaſtly remote æthereal regions: others,as if Comets were coæval to,and neither leſs permanent, nor more new, then the reſt of the Stars ; only ſeldome ſeen, and when ſeen, ſoon paſsing again out of ſight, by reaſon of the Line, upon which their Center is moved : and nothing as yet appears hindering the truth of the poſſibility and conſiſtency of both theſe Opinions ; leaving it eſpecially indifferent in the later Hypotheſis, to call ſuch ſtars at pleaſure by the name, either of Comets, or New-ſtars, or rather ſeldome appearing Stars. Comets upon the firſt ſuppoſition, ſeem not to want ſome affinity with ſeveral, eſpecially of the more eminent, meteors of our Atmoſphære: yet allowing a vaſt difference between them in place, proportion, duration, motion, and the like circumſtantials. And as our Globe, though in its ſelf large, is but a minute thing, compared with many of the cæleſtial Bodies; that the vaſt error of the Earths ſemidiameter, by taking the Earth for a ſmall point, many times in Aſtronomical, Gnomonical, and other operations, creates not by conſequence any perceiveable Errour: ſo it is not unlike, but the Atmoſphære of our Globe is vaſtly diſproportionable to the Atmoſphæres of many of thoſe heavenly lights: which may in a ſort be gathered alſo, from the ſmall power, which our Atmoſphære has in the preſſures which it makes upon moiſt and liquid bodies: as alſo from the common vaſtnes of cometical bodies, frequently accepted as much larger then all both our globe and its Atmoſphære, and therefore requiring a proportionately immenſe place both for their being and motions. And that ſuch huge dimenſions are not groundleſly attributed to the cometical bodies, is rendered undoubted from the glorious and glaring light, with which eminently above moſt of other Stars, they dazle, to admiration, the minds and Eyes of mortals at ſo great diſtance, that ſome times little, or no Parallax can be obſerved in them. As to theſe therefore to ſolve the *Que-*

ſtions, unto which you have tasked me; I cannot but judge it improbable, with any aſſurance to make prædictions before hand, that in ſuch limited moments of futurity, ſhall appear ſuch Comets, as are to be at after amaſſed, gathered, formed, and embodied in the Atmoſphæres, which are above: if ſeeming no more wilde to give heed and credit to the ſuperſtitious Auguries and ſoothſayings of the Antients, then the moſt ſubtle reaſonings upon which ſuch predictions can be made, in a ſubject of this nature. Yet though their firſt emerging and appearance cannot upon any calculation be foretold; however at after they have a while appeared, and ſome, though not very many, curious obſervations have been made of the changes of place, and other circumſtances, which they make in their ſeveral appearances, at ſuch and ſuch exalt diſtances of time; it is fairly enough probable, that often, by an ingenious prognoſtick, the courſe of ſuch aComet may be nearly traced, and ſet down before hand, as to way and ſpeed, only with this limitation, if the ſubſtance of the Comet do not firſt vaniſh, or diſſolve, or by reaſon of its inſtant diſſolution divert from, haſten, or retard its courſe: upon which ſcore of their diſſolubleneſs, and therefore their ere long to be expected diſſolution, no prognoſticks, upon them and their courſe, can be ſo abſolute and peremptory, as thoſe which may be made concerning ſuch Comets, as are properly couched under the ſecond ſuppoſition. And the grounds why moſt what in theſe Comets of the firſt conſtitution, or ſuppoſition, from paſt obſervations, a fair conjecture may be made of their future courſe notwithſtanding their new production and amaſſement in Atmoſphæres, are chiefly hereupon founded: that ſuch cometical bodies, as they are demonſtrably vaſt, and manifeſtly, and by their nature diſſoluble, ſo their diſsolution is not obſerved very momentaneous, but moſt what more, leiſurely by degrees and in time; in ſhort, they flame forth of a ſudden, but expire gradually; ſo as to be able, for a while, to retain the impreſſions of regular motion, whether communicated from ſomewhat in the Center of the ſame Atmoſphære, or from the confluent efficacy of ſeveral Agents all contributing conſtantly, each according to its nature, to the Comets alone, or its and its Atmoſphæres motion, whether by æquilibration, pulſion, traction, preſsure, manifeſt, or ſecret; or elſe inwardly wrought into the very ſubſtance and nature of the Comet: eſpecially the Comets place being in thoſe lofty æthereal regions, where are perpetual circumgyrations and every motion that is obſervable, the more accurately it is obſerved, it is found the more regular. And if it be urged here-againſt that the motion of Comets ſeldome, or never anſwers the motion of any other Starre, but commonly has a croſs trajecting motion, whereby it moves aſloope, athwart, tranſverſely, or obliquely, varying from the uſual, formerly obſerved motions. As this cannot be denyed, ſo it doth not at all infringe the poſſible regularity of ſuch motions, which are obſerved to be moſt what in the ſame plain, though oddly poſited: and that there are many and, diverſe ſuch undreamt of motions in the Heavens, conſtant and regular, of which ſave by ſome ſuch, or late diſcoveries, no hint hath as yet bin made forth unto us, may in as good reaſon be allowed, as that vaſt number of Stars in the skyes acknowledged, of which, without the Teleſcop, 'tis impoſſible to have any diſcerning. And if we grant that in reſpect of its peculiar Atmoſphære, the motion of a Comet ſolely conſidered as in it, be not ſo ſtrictly regular; yet the regular circumgyration of the Atmoſphære, in which the Comet is carryed, in reſpect of the Univerſe, cannot but with thoſe that Cant upon that Hypotheſis, enforce the Confeſſion of a great degree of regularity in the Comets motion: and that Comets, in what part of the Heavens ſoever appearing, move in ſome conformity with other celeſtial bodyes, notwithſtanding the contrariety or diverſity of their proper, or peculiar motions from the reſt, is, beyond all diſpute, evident from their dayly moving, with the reſt, from Eaſt to Weſt. The manner of making obſervations, calculating to know the place, the diſtance, ſpeed of ſuch a Comet, and when it is at neareſt, or remoteſt in its courſe from us, is no other then in Comets of the ſecond ſuppoſition. And from this whole diſcourſe is manifeſt; the Hypotheſes, upon which Calculations are to be made, in order to ſuch prædictions, cannot be particularly in theſe Comets known before hand, but in ſeveral ſuch Comets may be very different; and are to be found out by curious, and diligent Obſervations; which, in each reſpective Comet, muſt be accurate, and at leaſt four in number: and I think predictions pretended from three obſervations, are over haſty, and, at beſt, can but be conjectural; nay, in ſome caſes, from four; as if you be unſatisfyed, I ſhall be ready, by demonſtrative inſtances, to remove your doubt. But if of theſe Comets any ſhould be found, to wander, ſo as the motion of its Center cannot poſſibly be reduced any thing nigh unto the ſame plane, or regularity in change of Planes, or that the motion of its Center, in the ſame, or ſeveral Planes, deſcribe not a Line regular, or near unto a regularity, it is not probably eaſy to make predictions at any certainty of truth, of the future courſe and motion of ſuch Comets. Concerning Comets of the later kind, or Hypotheſis; poſſibly it is more eaſy to anſwer your Queries in them, then effectually without a world of time, ſagacity and diligence to accompliſh and attain the purport of thoſe Anſwers. That the motion of ſuch Comets of the ſecond ſort, being not truly new, but rather ſeldom appearing, and ſoon diſappearing Cometical Stars; that, I ſay the motion of ſuch, is not without ſome regularity, though it may not be concluded without long obſervations and frequent experiences of their motion had, which in ſuch ſeldom appearing, and ſoon again diſappearing Stars is difficult enough to attain to; yet if we conſider the ſteady, unbiaſſed courſe, held by moſt, if not all the reſt of the permanent Stars without deviation, though in different periods and with many different latitudes and declinations, and great diverſity of motions, it cannot be leſs then mightily probable, nothing in ſenſe, reaſon, or experience appearing as yet able to demonſtrate the contrary, but that a fair, celeſtial regularity may rationally be expected in thoſe their perpetual motions, whether their Centers be carryed in the ſame Plane, or in a continual change, and alterations of Planes. And in diverſe reſpects in ſome, if not all of them, as in moſt other Stars, the ſame Center may be

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ſaid to move regularly and perpetually in the ſame Plane, and alſo without elenchtical contradiction to move regularly with an inceſſant change of Planes: ſo the motion of any Planet, or other Starr, whoſe motion we are verſed in, taken concretely and with complication of that mundane, or general circumvolution, which with all the reſt it hath from Eaſt by South to Weſt, though it be regular, yet it is not without a continual ſhifting of the Planets, or Stars center from plane to plane: whereas the proper motion of the ſame Planet, or Starr conſidered abſtractly and ſevered from the general circumvolution it hath with the celeſtial univerſe, as it is regular, ſo it may keep the center of the planet, or Star conſtant in the ſame plane. And the Courſe, appearance, and returns of ſuch cometical Stars will be of much more eaſy calculation, if their motions, in the regularity, propriety and peculiarity of them, be any wayes reducible, as is aboveſaid, unto the ſame plane; of which plane by obſervations is to be decided, whether it paſs thorough the center of the Æquator and univerſe; and if the center of the univerſe lye out of it, what is the perpendicular from the Center unto the plane; and in the plane, what regular figure, or journey is deſcribed by the motion of the Comets center, and what are the Diameters or Axis of ſuch figures, or centers of ſuch motions in that plane: in the eaſieſt, and plaineſt of all which caſes, four obſervations are at leaſt re-requiſite: but where the regularity of the Comets motion, being irreducible to the ſame plane, is in a continual ſhifting from plane to plane, there many more obſervations may ſcarce be enough, to clear up the regularity of the Comets motion, ſufficiently for eaſy calculation: and when a ſufficient number of exquiſite obſervations have been made, the calculation, as it will hardly be ſo accurate, ſo it will ſcarce be ſo eaſy, as in other caſes- The obſervations, in all caſes and ſuppoſitions, are to make diſcovery of the exact place of the Comets center at the moment of the obſervations, *Viz.* of its longitude, latitude and altitude not in reſpect of Azymuthal arches, but in right lines drawn between their center and the center of the univerſe. And the more exquiſite the obſervations are, they make way for clearer calculations; upon which may be founded ſo much the more aſſured predictions. To anſwer therefore your *Queries* in relation to theſe Comets of the ſecond ſort, or ſuppoſition; I think it not impoſſible, but in time predictions may, it may be many Ages beforehand, be made of the Appearing, or rather Re-appearing of ſuch Comets: according as, in the regularity of their motion, they come nearer or are removed Farther off from us. So I remember a perſon of Quality in the North, with an high meaſure of Aſsurance reported to me, that one of the late Comets appearance and Emerging, was by a certain Genoeſe prædicted at leaſt two years before the Emerſion and appearing of it: which if ſo, is to be referred, I doubt not, to the principle and method, above inſiſted on. According to which, after different intervals of years, ſeveral Comets may appear, one after another, and ſometimes ſeveral ſuch Comets may appear all at once: and yet there is no clear neceſſity of the conſtant re-appearing of the ſame Comet, in the ſame Horizon, ſtill at after equal ſpaces of time. And if the Emerging of ſuch Comets may be prædicted before their Appearance; upon the ſame principles, the line of their motion, the caſting of their Train, and their future receſs, may as well be prædicted, whether before, or after their firſt appearance: as alſo when, and where ſhall be their Accelerations and retardations in motion; eſpecially, conſidering the advantage hereunto accruing, from their nearneſs to, or diſtance from the Eye of the ſpectator: not that this makes inequality, or inequability to be, but rather only to appear, in their motion. Upon the ſame accounts, the dayly riſing, ſetting, and Southing of Comets may be calculated: as alſo what parallel, or ſpiral it deſcribes: and how much it gains dayly in its proper motion: as alſo its place, as to longitude, latitude, altitude, above any limited Horizon, at any limite moment: what conſtellations it ſhall paſs thorough: and in what *Aſpect* it ſhall be poſited to any Planet, or other Star: ſo of conjunctions, oppoſitions, quadratures, combuſtions, Eclipſes, and many ſuch like affections, in common incident to Comets with other Stars, there may as clear predictions be made: as alſo when it diſappears, not for its remoteneſſe from us, but as other Stars dayly, becauſe of the Suns propinquity to it, and our view, the times of its not being ſeen by reaſon of the circumfuſion of the greater light of the Sun, being manifeſtly calculable; though of other obſcurations by clouds, and ſuch like impediments and interpoſitions, no artificial prognoſtick can be made. So in the firſt Approaches, or laſt receſſes of Comets, by falling into ſuch circumfuſions of ſolar light, the firſt appearance of the Comet will be later then otherwiſe, and in the other caſe, its appearing ceaſe ſooner then might be expected: and predictions are accordingly to be ordered. If I have been unuſually long beyond the proportion of a Letter, ſuppoſe it to be the long Train peculiar to ſuch Cometical, full-lighted, obſcure-natured Stars, as you are now Quæring about. And not to diſmiſſe you without a Task; it were a noble diſquiſition clearly to unfold, what it is in theſe Cometical Stars, alone, or alſo in their circumfuſed Atmoſpheres, that enables the Sun, more eſpecially to affix ſuch glorious, long luminous Tayls unto them more then to other Stars, whether conſtant, or new, or rarely appearing Stars; which in whatſoever poſition to the Sun, are ſtill ſeen without any ſuch long, beamy, luminous Trains. But I commit this and thoſe other *Queries* about the ſalts of the higheſt volatiles to your ſerious leiſure.

The ſixth Anſwer.

That the motion of the Celeſtial bodies about their particular axes, is not a ſatisfactory proof of their Copernican Syſteme. That Comets though ſuppoſed to be neareſt to the Earth, when they are in oppoſition to the Sun, yet do not thereby prove the Earths motion.

I See that neither the currency of Doctrines, nor the courſe of time, are able to beat off from you your old Philoſophical genius. You can ſwallow nothing upon the mere Tradition of others, nor yet upon their reaſons, till firſt examined, concocted, and digeſted in your own. Your doubts ſeem not cauſeleſs which you move upon that point; whether the motion of the Celeſtial bodies turning themſelves about, upon their own Axes, may be allowed for a juſt proof of the Copernican ſyſteme. It is a golden and noble branch of intellectual Juſtice, ſo far as is poſſible, mathematically and to a punctillio, to clear and ſet forth the bounders of Truth: yet as your modeſty delivers your judgement, I doubt not, by way of doubt; ſo I deſire you read my Anſwer, as writ only in the ſame Dialect; that I, in like manner, may not appear a raſh Judger, of what it ſeems has been concluded by men of great parts and learning. The Artfullneſs of the Inſtruments, and diligence of the Obſervers that, from ſeveral marks in ſeveral Stars, have noted this their circumvolution, I love and honour: and am apt to conjecture, the like motions may in future be diſcovered in other Stars, in which they have not as yet been noted: and ſhall be ready to improve the Obſervation in all inferences authorized and recommended unto us by the neceſſity of their conſequence. But as things, at firſt glance, commonly offer a fuller raye then they hold in conſtant; ſo I ſuſpect the glimmering probability of the Earths motion according to the Copernican Hypotheſis, upon the firſt diſcovery of this circumgyration of Stars about their own Axes, will, upon farther conſideration and deliberation, grow ſtill more faint. There being very few Hypotheſes, with which (to give it a ſhort name) this circumaxal motion of the Stars is expreſſely inconſiſtent; it not being impoſſible, if all things elſe would conſiſt and cohære, but ſuch a motion might be accompliſhed in the Hypotheſis of Orbes and Sphæres, ſo the Star were but unfixed and ſet lax and free to move according to its nature in its own orb, by a motion diſcharged, different and diſtinct from the motion of its orb: So in the old Hypotheſis, while they affixed and bound the Starre unto its proper Epicycle, yet in its larger Orb they allowed it to be therein in carryed ſometimes progreſſively, ſometimes retrogradely, and ſometimes ſtationarily, according to the motion of the Epicycle, or Epiſphære in the larger ſphære. And without any ſenſe as yet, that it deſerves a bluſh, I willingly profeſs, I want the Lyncean eyes and judgment, from ſuch a circumaxall moving of Stars, to deduce the Suns, or Earths central, either reſt, or motion: eſpecially the vaſt diſtance, at which theſe bodyes are diſtant one from another, being on all hands yielded; which if it do not neceſſarily, altogether amort, and aboliſh the powers, which one of theſe bodyes may poſſibly have upon the others, yet till ſuch their powers over one another in this point of motion be fairly proved, it leaves it eaſily manifeſt, that any of theſe bodyes may be moved according to its own peculiar nature, without laying, by ſo long a reach, laws of neceſſity upon other Bodyes, at ſo great a diſtance, for their motion. As for example the bodyes A. and B. either of the ſame general, or ſpecial nature; they being ſevered and ſeverally placed at competent diſtance one from another; both, or either may be at reſt, or in motion, neither inferring upon the other the analogy either of its reſt, or motion, or to be ſuſpected ſo to doe, till it be pregnantly either ſo obſerved, or proved: *Viz.* That either has upon the other, even at diſtance,

ſtance, ſuch a ſtrong and efficacions power. And if the ſtreſs of the ratiocination hang upon this; that otherwiſe there would not be a conformity between the motions of the Earth, the Sun, Saturn, Jupiter, &c. to my apprehenſion, nothing is more eaſy, then not only to turn the edge, but break the back and overturn the foundation of ſuch an Argument. For firſt the motions, or unmovednes of theſe bodyes, is not to be made conform unto our retired thoughts, and ſpeculations in a cloſe ſtudy, or to the motions, which we have obſerved, and juſtly concluded certain in other noble Bodyes in the world; but they are rather by all ingenions Arts and inſtruments, to be each by its ſelf particularly obſerved, how in Truth they are, and ſo to be accepted and acknowledged in their appropriate ſciences. And what greater neceſſity is there of making the Sun, or the Earth in reſt and motion to be conform to Saturn, Jupiter, &c. then to be conform to them in ſhape, magnitude, reſpective diſtances, which on no hand is aſcerted, but for the impoſſibility, and demonſtrablenes thereof by obſervation, rather the contrary? And why ſhould there not be as great a neceſſity for them to be conform in reſt and motion, to the Stars paſſing generally under the notion and title of Fixed, which has never yet been offered, becauſe it is clear it cannot be defended? And what leſs neceſſity is there, why they ſhould not conform in the celerity and nimbleneſs of their motion, as well as the form, kind and manner of it? And what greater neceſsity for conformity of motion in Stars, then in Animals, in which ſome creep, ſome fly, ſome go upon feet, ſome ſwim; neither in their ſuppoſed and called Elements can any ſuch neceſsity of the ſame kind of motion be obſerved, though in the ſame Element. Upon the ſame way, method and force of reaſoning, when *Jacob* is at the Dancing, or Fencing Schools in *Oxford*, *Joga* muſt be Galliarding or Brandiſhing it at the like Schools in *Salamanca*. Every ſtone has not in it the myſteryes of the Magnet, and ſeveral Jewels have their ſeveral ſplendours, virtues and properties: ſo ſeveral Stars conſidering their ſeveral ſituations, ſhapes, magnitudes, diſtances and diverſities of nature, cannot but in reſt and motion each have ſomething proper and peculiar: at leaſt may not without cauſe be ſuſpected to have otherwiſe, till it be poſsible by ſome medium to evince the contrary. And as the Copernican Syſteme receives not much confirmation from the præcedent circumaxal motion of Stars; ſo that Comets are neareſt the Earth when oppoſite to the Sun; with what greater ſtrength doth it, as others urge, prove the Earths motion? wherein is requiſite that the Hypotheſis it ſelf, be confirmed by frequent Obſervation in ſeveral Comets, before it be allowed as general; that as by way of Induction may be inferred, that every Comet when it is neareſt to the Earth, is then oppoſite to the Sun, and when this is allowed and fairly proved, there is no neceſsity by which it follows, or can be inferred from hence, that the Earth hath its annual Revolutions about the Sun: all that is in the allowed, and ſuppoſedly proved Hypotheſis, being many ways accompliſhable, and as eaſily and clearly, the Sun moving about the Earth, as the Earth moving about the Sun, in which I doubt not your judgment.

The Second
SECTION
of
PHILOSOPHICAL
ANSWERS.

CONTEINING

QUERIES

Relating to the Angle of Contact.

LONDON,
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UMI

To the right reverend Father in God, *Seth* Lord Biſhop of *Salisbury*.

To the right reverend Father in God Lord Biſhop of *Cheſter*.

My Lords.

Although I know, the raw, untrained Reader, upon this laſt Anſwer above the reſt, will be ſure to pronounce his, Quis leget hæc? *and to beſtow upon it, no other fate then* thuris & ſcrombrorum; *judging all things impertinent, that lye not in the dirty troden way; or cold, which his torpid dullneſs is unable, at leaſt too ſlothfull to ſtrike any fire out of; and every thing to be deſtitute of a kernel, which is too hard for his delicate Gum to crack the ſhell of: however, I fear not the rendering my ſelf faulty beyond Apology, in laying this Anſwer, above all the reſt divulged in this preſent manual, at your Lordſhips feet. The ſingular happines of both your Lordſhips in your ſtudies, drawing in by way of divertiſement, what multitudes of others ſtudy ſeriouſly and purpoſely for, and yet miſs of, having made veneration to wait upon your judgment,* *and your minds ſo ſeemingly fruitful, in multitudes of as new as ingenious inventions; hath too well acquainted your Lordſhips, with the Scufflings which have been in former ages, and between men of the cleareſt judgments, and moſt excellent parts, about theſe minute angles of contact; being, by general confeſſion, ſmall beyond all proportion with rectilinearies; and by others only denyed to be minute, becauſe they think they have no quantity at all. Like as in ſolar Atomes, when myriades of them are before us, we cannot be peremptory in aſſerting a full ſight of any one of them. And however I may have miſſed the* Corinth *I firſt hoyſed ſayl for, I know when retiring from your higher-region'd ſpeculations you are pleaſed to make unto your ſelves a Platonick divertiſement, upon a deigned peruſall of this ſcrole to a friend, I doubt not but your Lordſhips will juſtify the eſſay, and allow that by ſuch ſallyes, a firm comprehenſion may at laſt be attained of the mercurial niceneſs of this queſtion, which hath ſo oft eluded the graſp of thoſe wits, who have formerly thought they have had it ſo firm and ſure.* My Lords, *I know the admirable quality, and mighty merits of the perſons that think and have thought otherwiſe, then I have delivered my thoughts here; however I ſhould have dealt faithleſly with my friend, if I ſhould have returned him anſwer contrary to my judgmeut. And as I* *have been morally true to him, ſo what right, or injury I have herein done unto Philoſophical truths in this nice, and hitherto inexplicable ſubtilty, none can better decide then your Lordſhips; at whoſe feet it is left, by*

Your Lordſhips Servant.

W. M.

The Seventh Anſwer.

Concerning angles of contact. That recto-convexe angles of contact are truly angles according to the definition of plane angles. That recto-convexe angles of contact are not deſtitute of quantity. How many wayes one quantity may be greater then another, what kind of homogeneity it is that is requiſite for proportionalls. That angles have figuration as well as quantity. That in angles is obſervable a homogeneity or heterogeneity which is Mathematical, and alſo a homogeneity or heterogeneity which is not Mathematical, nor any way concerning their quantity, but only their figuration, &c. Of ſeveral ſpecial properties of magnitudes and angles. How ſome angles are Conſtituted by the concreſcency and annexion of ſeveral parts and angles which are heterogeneal each to other. Whether like angles in unequal circles, and in loke ſegments of circles be always Equal? That all Equal angles are not like. That all like angles are not Equal.

THough I cannot conſent unto your thoughts about the Angle of Contact, I ſhould be angry with my own raſhneſs, ſhould I be forward to blame your late ſpeculations thereabout; ſeeing ſo many noble and ſprightly wits, and ſuch as have deſerved the honour of *Europe*, ſiding with you. Eſpecially I honour your thorow-converſation in the learned perſons works, whom I perceive you ſo largely to have gleaned after: you could not have plowed in theſe hard grounds with a more worthy Aſsiſtant. But knowing it is not Complement you expect in Philoſophy but Reaſon; I ſhall in as ſhort, as the matter will bear, and as plainly as I can, ſumme up the grounds, why I as yet remain ſo farre diſſatiſfyed, that I cannot without injuſtice to my own judgment, deſert the generous and learned diſſenters from your Opinion: or ceaſe to averre Angles of Contact to be true quantities, equal; or unequal one to another, though the higheſt multiplex of the greateſt citradiametral Angle of Contact, be never able to exhauſt, or equalize the leaſt right-lined part of the moſt acute right-lined Angle.

And that we may not miſtake each other about the State of the Controverſy; and to be ſhorter, and more clear in the inſuing diſcourſe; we ſhall firſt ſet down the definitions of certain termes, about which I ſuppoſe there will be no difference between us. As.

1ſt. (Our diſcourſe extending no further then to plain Angles) that by a Right-lined angle, we underſtand a plane angle contained under two concurring ſides, or lines, which are two right lines not poſited directly the one in the production of the other, nor parallel the one to the other. As in fig. 1. the Angle BAC is a right-lined angle contained under the two right lines BA and CA; BA and CA not being ſo poſited at A as to make one right line: nor can they be parallel becauſe of their rectitude and concurrence.

2ly. That by a crooked-lined *A*ngle, we underſtand a plane Angle, contained under two concurring ſides, or lines, which are two crooked lines; as in the fig. 2. 3 4. the *A*ngle B*A*C is a crooked lined angle contained under the two Crooked lines B*A* and C*A*.

3ly. That by a crooked lined angle of concaves, or a Concavo-concave angle, we underſtand a plane angle contained under two concurring crooked lines; both which crooked lines obvert at the *A*ngle firſt of all inwards, the concave ſide of their Curvature, towards the angle under then contained. So in fig. 4. the angle B*A*C is a crooked-lined angle of Concaves, or a Concavo-concave angle, contained under the two crooked lines BA and C*A* both which obvert the Concave ſide of their curvature towards the angle under then contained.

4ly. That by a crooked-lined angle of Convexes, or a Convexo-convexe angle, we underſtand a plane angle, contained under two concurring crooked-lines; both which crooked-lines obvert at the angle firſt of all inwards, the Convexe ſides of their Curvature towards the angle, under them contained. So in fig. 2. the angle BAC. is a crooked-lined angle of convexes, or a Convexo-convexe angle, contained under the two crooked-lines BA and CA, both which obvert the convexe ſides of their curvature, firſt of all inwards, towards the Angle under them contained.

5^{ly}. That by a concavo-convexe angle we underſtand a plane angle, contained under two concurring crooked lines; the one of which, at the angle firſt of all, obverts the convexe ſide of its curvature inwards towards the contained angle, the other the concave ſide of its curvature. So in fig. the *3d.* the angle BAC is a concave-convexe, or a convexo-concave crooked-lined angle, contained under the two crooked lines BA and CA, whereof the one BA obverts the convexe ſide of its curvature towards the contained angle, but the other CA obverts the concave ſide of its curvature towards the ſame contained angle.

6^{ly}. That by a mixed lined angle, we underſtand a plane angle, contained under two concurring lines; whereof the one is a right line, the other a crooked line. So in fig. *5th. 6th.* the angle BAC is a mixed-lined angle, contained under the right line CA. and the crooked line B*A*.

7^{ly}. That by a Recto-concave angle, we underſtand a plane angle, contained under two concurring lines; whereof the one is a right line, the other a crooked line, having the concave ſide of its curvature obverted towards the contained angle, ſo in fig. 5. The angle B.A.C. is drawn a recto-concave angle, being contained under the right line C. A. and the crooked line B. obverting the concave ſide of its curvature towards the contained angle.

8ly. That by a recto-convexe angle, we underſtand a plane angle, contained under two concurring lines; whereof the one is a right line, the other a crooked line, having the convexe ſide of its curvature obverted towards the angle contained, ſo in fig. *6th.* the angle B A C. is drawn a recto-convexe angle, being contained under the right line C A. and the crooked line B A. obverting the convexe ſide of its curvature towards the containd angle.

9ly. That by an Angle of contact we underſtand a plane angle, contained under two concurring lines; which two lines are neither one and the ſame line produced, nor yet in their preſent poſiture and inclination can poſſibly cut one another, how farr ſoever either, or both of them be produced. So in fig. 7*th.* and fig. 8*th.* the angle B*A*C. being an angle of contact, if in either figure the crookd-line BA, be produced beyond the point of contact A, unto D, and in fig, 7*th.* the crooked-line CA, in figure 8*th.* the right line C A. be produced beyond the point of contact *A* unto E; in neither the line C*A*E can fal into the production of the line BAD, nor in either ſhall the line C*A*E cut the line B*A*D. *And*

10ly. From the figuration, poſiture and properties of the ſides of ſuch angles of contact; that by a recto-convexe angle of contact, we underſtand a plane angle being recto-convexe, as above defined, and being alſo an angle of contact. So in fig. 8*th.* if in the recto-convexe angle BAC, the right line C*A* touch and cut not the convexe arch B*A* the angle BAC is a recto-convexe angle of contact.

11ly. That, by a recto-concave angle of contact we underſtand a plane angle of contact, which is alſo recto-concave, as above defined. As in fig. the 8*th.* if in the recto-concave angle E A B the right line EA touch and cut not the concave arch BA, the angle EAB is a recto-concave angle of contact.

12. That by a concavo-convexe angle of contact we underſtand a plane angle of Contact, being alſo concavo-convexe, as above defined. So in fig. 12*th.* if R*A*H being a concavo-convexe angle, the arch RA touch and cut not the arch H*A* then the angle R*A*H is a concavo-convexe angle of Contact.

13. That by a Citra-diametral-concavo-convexe angle of Contact, we underſtand a plane concavo-convexe angle of Contact, both whoſe conteining *Arches* lie on the ſame ſide of the right line tangent, which toucheth both the arches in the ſame point of Contact, *As* in fig.

12th. the concavo-convexe angle of Contact H*A*R, having both *A*rches H*A* and R*A* lying on the ſame ſide of the right line tangent B*A*, which toucheth both the *A*rches in the ſame point *A*, the concavo-convexe angle of Contact H*A*R is citradiametral.

14. That by an ultra-diametral concavo-convexe angle of Contact, we underſtand a plane angle of contact being concavo-convexe, and having the one arch on the one ſide of the diameter paſſing thorow the Contact point, and beyond the right line tangent which toucheth the *A*rches in the ſame point; but having the other arch on the other ſide of that diameter, and on the other ſide of the right line tangent. So in fig. 12*th.* the two Circles F*A*H and *A*DK touching in the point *A*, and AK being their common diameter and AB being a right line tangent to both, the concavo-convexe angle of Contact DAH is ultra-diametral.

15. That by a mixed crooked lined angle, we underſtand a plane angle contain'd under two concurring crooked lines, of ſuch ſeveral curvatures that it is impoſſible for them to be coapted from the angular point the one unto the other, or the one, or its production to be coapted to the other, or its production, at and from the angular point, by any circumduction whatſoever, ſo in fig. 9. 10. 11. the angle B*A*C

B*A*C, being a mixed crooked lined angle, AB as being of a different curvature from AC, can no way poſſibly be coapted by any circumduction to AC from the angular point A : and if B*A* be produced to D and CA to E the crooked-line AB, becauſe of its different curvature can neither be coapted to the crooked line AC nor to its production *A*E: nor the crooked line *A*C to the crooked line *A*B, nor to its production *A*D, by any circumduction upon the angular point whatſoever.

16. That by the iſoclitical ſides of an *A*ngle, we underſtand two ſuch lines, whether right, or crooked, containing a plane angle, as according to their preſent ſite and poſiture, without inverſion when crooked, moving by the ſides nearer one to another, will by ſuch motion at laſt come to be coincident with and coapted exactly the one unto the other: and that by an iſoclitical angle we underſtand a plane angle, contained under ſuch iſoclitical ſides. So every right lined angle is manifeſtly iſoclitical; and its ſides are iſoclitical, the ſides by ſuch motion on the angular point one towards another being manifeſtly coincidible and coaptable So alſo in fig. 3*d*. if in the concavo-convexe crooked lined angle BAC. the crooked-lined ſide B*A* by moving upon the angular point *A* towards the other crooked lined ſide CA, will at laſt coapt and become coincident with the croo-

ked lined ſide C*A* then are B*A* and C*A* iſoclitical ſides and the angle B*A*C aniſoclitical angle.

17. That by the aniſoclitical ſides of an angle we underſtand two ſuch lines, whether the one right & the other crooked, or elſe both crooked, and whether both of the ſame or different curvatures, containing a plane angle, ſo as according to their preſent ſite and poſiture, without inverſion of either when crooked, by moving the ſides nearer one another, by the continuance of ſuch motion they can never be brought to be coincident with, and coapted exactly the one unto the other and that by an aniſoclitical angle, we underſtand a plane angle contained under ſuch aniſoclitical ſides. So all crooked lined angles of concaves, though having ſides of like curvature, all crooked lined angles of convexes, though having ſides of like curvature, all concavo-convexe angles, whoſe ſides are of different curvatures, all mixed lined angles, whether recto-convexes, or recto-concaves, all mixed crooked lined angles whatſoever, are all of them manifeſtly aniſoclitical angles, and their ſides aniſoclitical. So in fig. 4*th*. though AB and AC. be ſuppoſed to be of uniform anſwering and equal curvatures, and likewiſe in fig. 2*d*. though AB and AC again ſuppoſed to be of uniform, anſwering and equal curvatures, and in fig. 3*d*. fig, 5,6,9,10, 11. ſuppoſing *A*B and *A*C not to be of

uniform, anſwering and equal curvatures, by moving the ſides AB & AC in that their preſent ſite & poſiture upon the angular point A, without the inverſion of either, when both are crooked, to bring the ſides nearer the one to the other, it is manifeſt that by the continuance of ſuch motion they can never be brought to a coincidence with, and to be coapted exactly the one unto the other, and ſuch alſo are all angles of Contact whatſoever, as in fig. 7, 8, 11. AB & AC are as uncoincidable and uncoaptable as in the former, except only ultradiametral, concavo-convexe angles of Contact, which are of equal and anſwering curvatures.

18. That by an Angle of curvature, or coincidence we underſtand a plane angle, contained by any two parts of a crooked line at the point of their concurrences any where to be imagined, or taken in crooked-lines. So in the 7*th.* fig. let BAD be the circumference of a Circle, Parabola, Hyperbola, Ellipſis &c. At the mean point A, the ſides BA and AD contain an Angle of Curvature, or coincidence.

19. That by autoclitical curvature, and ſo by an autoclitical crooked line, we underſtand ſuch a crooked line, as paſſing from the angular point of a right lined angle, between the two ſides, by the inclination of its curvature,

ture, keepeth the convexe ſide of its curvature, conſtantly obverted to one of the ſides, and the concave ſide of its curvature conſtantly obverted to the other. *As* in fig. 17. in the right lined angle BAC, the crooked line *A*EF keeping its convexity conſtantly obverted to the ſide AB, and its concavity to the ſide AC, or ſo much of it as is intercepted between the interſection at F and the angular point at *A* the crooked line *A*EF is autoclitical, *i. e.* the convexity is all on one ſide, and the concavity all on the other.

20. That by antanaclitical curvature and ſo by an antanaclitical curve line, we underſtand ſuch a crooked line, as paſſing from the angular point of a right-lined angle, between the two ſides, by the inclination of its curvature, hath the convexe part of its curvature, ſometimes towards the one ſide of the right lined angle, and ſometimes towards the other ſide of the right lined angle, *i. e.* the convexeneſs and the concaveneſs are not conſtantly on the ſame ſeveral ſides. *As* in fig. 17. in the right lined angle BAC, the crooked line AGH obverting the convexity at G towards AB, and at H towards AC is antanaclitical.

Theſe definitions præmiſed, to give now the true State of the controverſy; let there be, as in fig. 12. two equall Circles AHH and ADK touching in the point A: and let AG be the Semidiameter of the circle ADK: and AB be a right line tangent touching both the Circles in the point A: and let *A*EL be a greater circle then either, touching both the former, and alſo the right line tangent in the point A: and from the point A draw the right line AC at pleaſure, cutting the circle *A*DK in the point D, and the circle AEL in the point E: now therefore whereas you ſay that the Recto-convexe angles of Contact BAE and B*A*F are not unequal, and that neither of them is quantitative; and that the crooked-lines E*A*, DA, HA are coincident, *Sc.* ſo as to make no *A*ngle with the right line *A*B, or one with another; and that the right-lined right angle B*A*G is equal to the mixt-lined, *i. e.* Recto-concave angles of the Semicircles E*A*G and D*A*G ſeverally, and that thoſe angles of the Semicircles E*A*G and D*A*G are equal the one unto the other; and that the mixed lined Recto-convexe angle F*A*G is ſeverally equal to all, or any of the former; and that there is no heterogeneity amongſt plane angles, but that they are all of them of the ſame ſort, and homogeneal and undevideable into parts ſpecifically different, diſtinct and heterogeneal in reſpect of one another, and the whole; and that to any mixt, or crooked lined angle whatſoever that is quantitative it is not impoſſible to give an equal right lined angle : I acknowledge for all theſe things you have diſputed very ſubtilly ; yet I muſt with a clear and free judgment own and declare a diſſent from you in them all, for the reaſons to be alleadged in the inſuing Diſcourſe.

To clear all which , nothing can be of higher conſequence in this Queſtion, then truly to underſtand the nature of an Angle , what an Angle is, what is an Angle , and what is not. And to exclude the conſideration of Angles herein unconcerned ; they are plane angles *i. e.* ſuch as are contained by lines , which lye wholly , and both in the ſame plane , the diſquiſition of whoſe nature we are now about. And ſuch is the affinity , which the inclination of one line hath to another in the ſame plane, with the nature of an Angle , that without it a plane angle cannot be defined , or conceived. For where there is rectitude and voidneſs of inclination , as in a right line and its production, there never was juſtly ſuſpected to be any thing latent of an angular nature ; nor between parallels, for want of mutual inclination. But yet the inclination of line upon line in the ſame plane, is not ſufficient to make up the nature of an Angle : for in the ſame plane one line may have inclination to another , and yet they never meet , nor have in their infinite regular production any poſſibility of ever meeting ; as the circumference of an Ellipſis, or circle to a right line lying wholly without them, without either Section, or Contact; or the *Aſymptotes* in conjugate Sections, which though ever making a cloſer appropinquation to the circumferences of the conjugate figures, yet infinitely produced, never attain a concurrency. And though by poſſibility the inclined lines might meet, yet if they do not, an Angle is not conſtituted: only there is poſſibility of an Angle when being produced they ſhall meet. So as to make up the nature of an Angle two lines muſt be inclined one to another, and alſo concurre, or meet to contain on their parts a certain ſpace, or part of the plane between their productions from their point of concurrence, or their angular point. And this being the general, and proper nature of a plane angle; as it is manifeſt, that in a right line there is no Angle, ſo it is dubious what is to be judged of thoſe curve, or crooked lines, whoſe curvature is either equall, uniform, or regular: *Sc.* whether in ſuch lines there be not at every mean point an angularneſs, the lines ſtill lying in the ſame poſture. It is clear in a right line no mean point can be taken, at which the parts of the right line in their preſent poſition can be ſaid to have any inclination one to another: that though the parts concurre, yet they want inclination: but in the circumferences of circles, Ellipſes, Hyperbola's, Parabola's and ſuch like curve, or crooked lines, being of equal uniform, or regular curvature, no point can be aſſigned at which the parts in their preſent poſition have not a ſpecial inclination one to another. So as, as before, inclination of lines and concurrency making up the nature of an angle; it ſeems not reaſonable to deny angularity at any point of ſuch crooked lines, however their curvature be either uniform, equal, or regular. But I know it will be ſaid that ſuch crooked lines of equal, uniform & regular curvature are but one line, and therefore, by the definition of an angle, cannot contain an angle; which requires two lines to its conſtitution. To which I anſwer, that in like manner, two concurring right lines may be taken for one line continued though not in its rectitude, and then the conſideration of angularity between them is excluded: however by reaſon of the inflexion and inclination at the point of incurvation, there is an aptitude in that one produced line, there to fall, and diſtinguiſh it ſelf into two, with inclination of one unto the other, and ſo to offer the conſtitutive nature of an angle: and ſo it is at every point of ſuch crooked lines, whoſe curvature is equal, or uniform and regular; being one, or more lines, according as by our conceptions they are continued, or diſtinguiſhed

And as a continued rectitude, ſuch as is in right lines, is moſt inconſiſtent with the nature of an angle, ſo what ſhould be judged more accepting of the nature of angularity, then curvature is, being thereunto contrariouſly oppoſite, and by all confeſt moſt what to be ſo; ſaving in the aforeſaid caſes when curvature is equal, uniform, or regular. But why ſhould equality, uniformity, or regularity of curvatures, be ſo urged in the concern of angularity, by none of which is angularity either promoted, or hindered? For there cannot be greater equality, uniformity, or regularity, then is to be found in Rectitude as well as angles, in right lines as well as Circles; the things conſtitutive of the nature of an angle, being things quite different from them, *Viz.* inflexion or inclination and concurrence; which are indifferently found in all curvatures equal and unequal, uniform, or not uniform, regular, or irregular: and beſides are inſeparable from them: curvature and a concurrent inflexion, or angularity being but as two notions of the ſame thing, conſidered as under ſeveral reſpects: *Viz.* curvature is the affection of a line conſidered as one: the ſame being angularity and inclination, when from any point of the curvature the ſame line is conſidered as two: ſo two concurrent right lines are ſaid to be one crooked line: but when a right-lined angle is ſaid to be contained by them, they are then conſidered as two from the point of their incurvation, or inclination: and equality, and uniformity, and regularity of curvature implying equality, uniformity and regularity of inflexion, or inclination, it is ſo far from concluding againſt angularity, that it inferrs it with an additament, *viz.* of equal angularity, uniform angularity, and regular angularity; as might be at large declared in the ſpecial properties of ſeveral crooked lined figures. And in ſeveral uniform, and regular crooked lined figures, there are ſome ſpecial points, offering even to view and ſenſe a clear ſpecimen of a more then ordinary angularity, without any ſuch loud calling for the ſtrong operations of the mind; as the vertical points in conjugate figures, in parabola's, and the extream points of either axes, in ellipſes, and the like, And in the definition of a plane angle, all the inclination, which is required in the concurring ſides, is only, that their inclination be μὴ ἐπ' εὐθείας *i. e.* ſo as the ſides make not one and the ſame right line, as it is generally underſtood: however ſo far are we from impoſing upon any againſt their judgements, this ſpecial ſort of angles, that we readily acknowledge, if the words μὴ ἐπ' εὐθείας, in the definition of a plane angle, be forced from the commonly received ſenſe, of the ſides not lying in the ſame right-line, to ſignify the non-coincidency of the production of the one ſide with the other, then according to that gloſs upon the definition of plane angles, this whole ſort of angles is to be rejected; wherein every one is freely left to his own judgement; the difference being a queſtion and quarrel about words, more then matter, and not concerning the preſent controverſy.

However from the whole it out of Controverſy appears, that recto-concave angles of Contact are true angles, contained under two inclined ſides, concurring: alſo that ultradiametral concavo-convexe angles of Contact, whether leſs equal, or greater then two right right-lined angles are true angles, for the ſides are inclined and concurr: of their concurrence can be no doubt: and that they are inclined muſt of neceſſity be yielded, ſeeing they neither lye in rectitude, nor which to ſome might be a cauſeleſs ſcruple, the one in the production of the other: ſo as their angularity is clear beyond all doubt: the inclination underſtood in the definition of an angle being generally any poſiture of one line with regard to another in the ſame plane, ſo as both neither fall in one right line, nor be ſituated parallel one to another, nor under ſuch a manner of oblique extenſion, as may render their concurrence impoſſible: it is not only the oblique poſiture of one line in reference to another, as contradiſtinguiſhed from perpendicularneſs in acute and obſuſe inclinations: but ſuch is the comprehenſiveneſs of its ſenſe, in the preſent acceptation, that every perpendicularneſs it ſelf is taken for an inclination.

So likewiſe it cannot reaſonably be denyed but thoſe ſpecial angles of Contact, which are the chief ſubject of this preſent controverſy, I mean recto-convexe angles of Contact, are truly angles; except either the inclination of their ſides, or their concurrence could be called in queſtion; nothing elſe being requiſite unto the nature, or comprized in the definition of an angle: and the like is to be judged of all other concavo-convexe and convexo-convexe angles of Contact. But you ſaid that in the recto-convexe angle of Contact the right-line tangent and circumference make no angle, becauſe the tangent is not inclined to the circumference, but coincident with it. What myſtery of reaſon, or force of *A*rgument ſhould be in this deduction? if you ſay it is coincidence, you mean by non-inclination; I readily yield, where two lines, become coincident, their former angle is thereby extinct; as thereby they come under the conſideration but of one line: as when two right-lines, or two iſoclitical crooked lines, are, one of each of them, ſo moved about the angular point, till the two lines become one. But where is any ſuch coincidency between right-line tangents and circumferences? or what poſſibility is there of any ſuch coincidency: a crooked line and a right-line may no doubt be commenſurate, or proportionable in length, but in poſition it is impoſſible: and if we imagine the tangent bowed to ſuch a coincidence, then it is not any longer a tangent, or right-line, but a circumference. And though as you urge, a right-line circumduced about any middle point in the ſide of a regular polygone, at laſt becomes coincident with the ſide, and looſes all inclination and angularity with the ſide; what doth that concern, or how doth that prove the non-inclination of the tangent to the circumference; your ſelves ſometimes in every point, ſave only in the point of Contact acknowledging an inclination; and as is elſe where hinted, in the point of Contact alone, and abſtractedly, the inclination it ſelf, which is the habitude of the concurring inclined ſides, is not to be ſought, but only the particular termination of their inclination there. But you will ſay, it is non-ſecancy which is meant by this coincidency: and what I pray is that more then the Lateran bells to the concern, or conſtitution of angles ? Are there not many regular curve lines produced , ſome infinitely without Section , in which eſpecially the circumferences of circles , you are pleaſed ſometimes every where to think you have cauſe to imagine an angle. It were meet to know the meaning of ſuch odly connext terms , before reaſonings upon them be regarded in queſtions of weight.

That the Contact of the angular ſides , is as different from coincidency , as from ſecancy , is moſt unqueſtionably apparent in angles of ſuch ſides , as are capable of all three , *Viz.* Contact, ſecancy , and coincidence. For example in fig. *16.* let there be two Arches of equal Circles DAF and BAC. touching one another in the point A. If the arch DAF be circumduced about the point A as an unmoveable center ; at after an infinite ſucceſſion of ſecancies , at laſt all will terminate in a manifeſt coincidency , and the arch AF be coincident with the arch *A*B , and the arch AD be coincident with the arch AC. So as ſecancy , Contact , and coincidency are diſtinguiſhable one from another with as much eaſe and cleareneſs , as an odde number from an even. But if it be urged, that you aſſert not a coincidency between the arch. D*A*F and the arch BAC , but only that GH being a right line and touching the arch B*A*C in the point *A* , that I ſay the right line

GH and the arch BAC are coincident. The vanity of this may be evinced; in that by the ſame reaſon it followes, that the right line GH muſt be coincident with the arch DAF, and ſo the arch DAF coincident with the arch BAC; the contrary of which is above-ſhewn and confeſſed: and beſides hereupon ſhould two arches BAC and DAF being convexo-convexely poſited, and the right line tangent GH be all coincident; which I leave for others to ſay rather then my ſelf.

When a right line tangent and many crooked lines of different curvatures, all touch together in one and the ſame point, as in fig. 13; you ſay, though without the angular point of Contact the ſides are variouſly divaricated one from another, yet in the point of Contact, they have not ſeveral inclinations, for you ſay they have no inclination at all. The truth is the angular point of its ſelf is not capable of inclinations, nor for the indiviſibility of its nature can by any poſſibility comprehend them: yet that lines concurring in one only point, and preſently after receding each from the other, ſhould not be inclined each to other in or at the point of their concurrency, whether it be by Contact, or Section, though in the caſe of Contacts the inclination be leſs, then can be expreſſed by the inclination of any right-line upon a right-line, is abſolutely unconceiveable, there being no lineary coincidence, but only of one point between them. For as elſe where, in the ſame plane, neither of point to point, nor of point to line, nor of point to plane can be any inclination: but in the preſent caſe of plane angles, inclination muſt be of lines, and may be of them in the very point of concurrence; or elſe from the point of concurrence they would not part ſeveral wayes: for it is their diverſe inclination at that point, which makes their departure one from another when they depart from thence. And as even in the angular point of right-lined angles the lines have the ſame inclinations as elſe where, ſo in all other angles, ſave only ſuch angles of Contact, as are leſs then the leaſt right-lined angles, by drawing right-lined tangents to the arch, or arches at the angular point, is ſhewn in right-lines, either the very inclination of the ſides, in, or rather at the angular point, or elſe the leaſt right-lined inclination; which is greater, or the greateſt right-lined inclination, which is leſs: for though they may differ much in their diſtance and divarication one from another, without the point of their concurrence; in the point of their concurrence, without much abſurdity, they may be ſaid to be equally diſtant, *i. e.* not at all diſtant there one from another, for there they are not indeed at all diſtant any of them from the reſt: yet it doth not hence any way follow, that in like manner it may be ſaid of them without abſurdity, that in the point of concurrence they have equal inclinations, *i. e.* no inclinations one to another: for though in the point of concurrence it is truth, they have no diſtance, yet it no wayes as may appear to thoſe that will conſider equally follows, that they ſhould there have no inclination.

Beſides, that the urging of the coincidency of the ſides in recto-convexe angles of Contact is moſt directly oppoſite to the nature and properties of the ſpecial lines under which ſuch angles are contained. For, tis the ſpecial propertie of ſome lines that they can touch, but they can no way be-coincident; as Arches of unequal and unlike curvature: and a right-line and a crooked line: ſome can be coincident, and can no way touch, as right lines: ſome can both; as Arches of equal and anſwering curvature, which ſet concavo-convexely, and citradiametrally cannot touch, but will fall into a coincidence: but poſited convexo-convexely, cannot be coincident, but may conſtruct an angle of Contact.

So as the chimæra of the coincidency of the ſides in recto-convexe angles of Contact, if perſiſted in, is worth laughing at, and like his Philoſophy who when every one was at his high Lavolta's, denyed the poſſibility of motion in the world: But to juſtify the non-inclination of the ſides againſt the eye and reaſon, this horridly diſtorted Monſter of their coincidency was introduced. Indeed if they be coincident they make no angle: But it will cramp the underſtanding of an *Oedipus* to declare how either a right-line, or a crooked-line, touching another crooked-line in one only point and no more, ſhould ever be conceived, notwithſtanding to be coincident with the production of the other crooked-line, whether the tangencie of the crooked-lines be concavo-convexe, or convexo-convexe, *i. e.* the one within the other, or elſe the one turned away from the other. But you'll ſay, you aſſert coincidency only in the point of Contact. I anſwer, that's frivolous, not to ſay ridiculous and impertinent; for coincidency in the preſent queſtion of angles is taken as oppoſed to inclination, which is an affection and propriety of the concurring ſides of the angle, not only of the angular point taken by it ſelf abſtractedly. As inclination cannot be in a point, though it may be at a point, ſo a point cannot be ſaid to be inclined unto a line, eſpecially it ſelf being in the ſame line: it may be ſaid to hold ſuch and ſuch a diſtance from the line, when it is without it, but not to be inclined unto it. And if the being of the point of Contact , generally as a point of the one ſide , in the tangent line , as in the other ſide , whether right , or crooked , make a coincidency deſtroying inclination ; then all inclinations and angles whatſoever are deſtroy'd ; and every where will be a coindicency , for that is common to every angle to have the angular point ſtill common to both ſides : and the ſecant angles might as well be ſaid coincident , as the tangent angles. For what you ſay , that it is tangency as oppoſed to ſecancy, that you mean by coincidency ; I anſwer the gloſſe is improper and beſides the anvil and tangency undenied , but in this caſe impertinently by you alleadged , till it be proved that tangency in one only point , and no more , doth quite annul and deſtroy the inclination of the lines , though on both ſides of the Contact clearly receding the one from the other : till which be done happineſs to my friend and no longer.

I might adde that it is Touching which is only mentioned in the definition of plane angles ; but I ſhall diſpatch all by ſetting the caſe before you in this Diagramme in fig. 12. if the right line BA touch the arch LA in the point A and the right-line LB be ſo drawn as to touch the arch LA in the point L, here now is a plane on every ſide bounded by two right-lines , *Viz.* BL and BA and one crooked-line, *Viz.* the arch LA. That LBA is an angle will not be doubted: and becauſe the three lines perfectly bound in and limit a plane on all ſides; the Arch LA can neither be coincident with the right line LB, nor with the right-line AB, and in the angle LAB, the lines LA and BA; and in the angle ALB, the lines LA and BL concurring without coincidency, and without lying in the ſame right-line, or the one, ſo much as, in the production of the other; inclination, and ſo the true nature of an angle, cannot be denyed them; according to the moſt ſevere limitations and hardeſt gloſſes, that can with any reaſon be deduced from the definition of a plane angle. And to make all clear let a Paper, or other plane be cut in the form of the mixed-lined Triangle ALB and the wildneſs of queſtioning the angularity of the two recto-convexe angles of contact LAB and BLA will be clear to all perſons both rude and learned. I take it therefore for granted; that, all ſuſpicion of coincidency and non-inclination, in whatſoever pertinent ſenſe, of Contact-angle ſides, being evicted and ſent of the ſcene, all recto-convexe angles of Contact are truly angles.

To paſſe now unto another of your Theſis's in which you peremptorily conclude Recto-convexe angles of Contact to be devoyd of all quantitativeneſs ; when I urge their quantitativeneſs, I mean not that they can at pleaſure be devided into parts in any given and limited proportion, or by a mathematical homogeneity holding any proportion with the Angle of Contact devided; only that from the angular point between the ſides of the leaſt Recto-convexe angle of Contact infinite other lines may be drawn, dividing the angle, though heterogeneally. And certainly its being an angle of Contact cannot in the leaſt prejudice its quantitativeneſs: for it is moſt apparent that a convexo-convexe angle of Contact contained under *Arches* of equal curvature, may be divided into two mixed-lined, *i. e.* recto-convexe angles of Contact, which hold proportion of equality one with another, and each of them is in ſubduple proportion to the convexo-convexe angle of Contact, which was divided: and infinite numbers of Contact-angles of ſeveral ſorts may be adjoyned one unto another, diſtinct in their ſituation, without drowning and extinguiſhing one another and each lying without the other: which is not true of indiviſibles when they are adjoyned to, and touch one another. Beſides the recto-concave angle of Contact is greater then the greateſt right-lined angle, at the angular point of Contact having a manifeſt inclination of the ſides; for concurring, they neither are one right-line, nor one crooked-line. Moreover ultra-diametral concavo-convexe Angles of Contact may be equal to two right right-lined angles, or greater ſometimes: *A*s in fig. 12. if the two Circles FAH and ADK be equal and touch in the point *A*, and AK be diameter: and AR be another *A*rch falling beyond the diameter *A*K and beyond the right line tangent AB: it is manifeſt that the concavo-convexe ultra-diametral angle of Contact DAH is as to the receſſe of the ſides equal to two right-lined angles: for the recto-concave angle of Contact BAH is equal to two right right-lined angles, deducting the recto-convexe angle of Contact HAS: and the recto-convexe angle of Contact BAD is equal to the recto-convexe angle of contact H*AS*; therefore the ultradiametral concavo-convexe angle of Contact D*A*H is equal to two right right-lined angles: and therefore the ultra-diametral concavo-convexe angle of Contact D*A*R is greater then two right right-lined angles.

And then, what greater Monſter is diſcoverable in the doctrine of the quantitativeneſs of the recto-convexe angle of Contact? it is demonſtrated that between the right-line tangent and the Arch which it toucheth, no right-line can paſſe: *i. e.* the recto-convexe angle of Contact is leſs then the leaſt right-lined angle: but why ſhould hence be inferred that the recto-convexe angle of Contact hath no true quantity? you will ſay, becauſe a right-lined angle is infinitely diviſible into leſs and leſs parts, and therefore muſt at laſt be leſs then the recto-convexe angle of Contact, if the recto-convexe angle of Contact be a true angle, having truth of quantity. I anſwer, after all poſſible diviſions of a right-lined angle into parts of its owne kind, I mean ſuch as are made by right lines, the leaſt part is ſtill a right-lined angle; then which the recto-convexe angle of Contact is moſt fairly demonſtrated to be ever leſs: *i.e.* the inclination of its ſides is ever leſs: and the convexe arch will ſtill at the angle fall within: ſo as truly from thence may be inferred, that the recto-convexe angle of Contact can never be either a right-lined angle, or equal to it, or greater than a right-lin'd angle: but that therefore it is no angle, and hath no true quantity at all, becauſe it hath not the quantity of a right-lined angle, is a wild and perverſe inference, and elſe-where diſproved.

And why ſhould the quantitativeneſs of the recto-convexe angle of Contact be called in queſtion, becauſe it is demonſtrated leſs then the leaſt right-lined angle? If between the right-lined tangent and the arch a right-line could be drawn, then you would confeſſe the quantitativeneſs of it as undeniable: and why doth not the paſſing of a thouſand crooked-lines from the angular point, between the ſides of the recto-convexe angle of Contact, as well prove its quantity and diviſibility, as the paſſing of one right-line between them could: there being equal force of proof from the one, as from the other? And when a right-line & two equal Circles, all three touch in the ſame point; there are two equal recto-convexe angles of Contact adjacent the one continuedly to the other, and ſituate the one without the other: which in indiviſibles is impoſſible. And when the queſtion is of the quantity of angles, what is it we enquire, but only, what is the inclination of the ſides, eſpecially at the angular point? And in recto-convexe angles of Contact, the anſwer is; there is no inclination at all as of a right-line to a right-line, but only as of a crooked-line to a right-line: that it were wildneſs to ſay; becauſe in recto-convexe angles of Contact, there cannot be the inclination of a right-line to a right-line, that therefore the ſides meeting and parting one from another, and not lying both in a right-line, do make no inclination one to another.

And ſeeing the convexo-convexe angle of Contact contained under two Arches of equal curvature is dividable into two equal parts by a common right-line tangent to them both;

angle of Contact, and of both the recto-convexe angles of Contact, clearly appears; though the quantitativenes of every one of them be demonſtrable to be leſs then the quantity of any the leaſt right-lined angle whatſoever: becauſe right-lines cannot contain a leſs angle then is agreable to the inclinations they poſſibly can have one to another: whereas in crooked lines generally, theſe are the conſtant properties of curvature; though the angles of coincidence, or curvature, as we for method, brevity and diſtinction ſake named them, may be very unequal one to another, according to the degrees of the inflexion, inclination and curvature; yet conſtantly the angle on the concave ſide, from any point of the curvature manifeſtly if it be equal, uniform and regular, is ſtill greater then any the greateſt acute, right, or obtuſe right-lined angle: and the angle between the convexe arch and a right-line tangent, at the ſame point, is conſtantly leſs then any right-lined angle: and yet either may be made ſtill infinitely leſs, or greater, the recto-convexe angle of Contact ſtill remaining leſs then the leaſt right-lined angle, and the angle of curvature, or coincidence greater ſtill then the greateſt right-lined angle: which is as much as to ſay, that in curvatures the difference between the angle of curvature and two right right lined angles, cannot be a

right-

right-lined angle; as in truth it cannot, nor is in reaſon ſo to be expected, but of neceſſity it muſt be a mixed-lined, *Sc.* a recto-convexe angle: and is the recto-convexe angle of Contact at the ſame point. And that a recto-convexe angle of Contact, by no multiplicity can equal, or exceed a right-lined angle; doth not diſprove either its angular nature, or its quantitativeneſs, both which are otherwiſe cleared, but it is rather a confirmation of the heterogeneal difference, which is between the angles of the one ſort and the other. And in my judgement there needed no greater argument of the quantitativeneſs of recto-convexe angles of Contact, then the abſurdity following upon the contrary doctrine; that the angle of a ſemi-Circle, and a right right-lined angle are equal, *Viz.* The whole and a part, the one being a mixed-lined and the other a right-lined angle; and in the indeavour of coaptation, and being coapted on one ſide, the other ſide all the way falls within, or without the other, ſo as both the ſides of the one angle are impoſſible to be coapted to both the ſides of the other, but will both ſtill lye within both the ſides of the other. And the angles of ſemi-Circles muſt either be confeſſed unequal in unequal Circles, or the curvature of unequal circumferences, be manifeſtly againſt the truth aſſerted to be equal. And if ſtill they be averred to be equal, its deſired their equality ſhould be demonſtrated, and the way of admeaſuring their equality ſhown.

But you will ſay, if recto-convexe angles of Contact be quantitative, why can they not exhauſt any other angle whatſoever contained under the very ſame ſides; for poſſibly you will urge, that we ſhould not queſtion the homogeneity between ſuch angles? To this I anſwer, without examining what homogeneity may in other reſpects be between them; that a quantitative and mathematical homogeneity can with no reaſon between them be imagined: becauſe the difference which is between them is a right-lined angle, to which all angles of Contact whatſoever are heterogeneal; and your ſelf will not aſſert any mathematical homogeneity, or at leaſt proportionablenes which as to this purpoſe is all one between any recto-convexe angle of Contact and a right-lined angle. As in fig. 18. let *K*AB be a recto-convexe angle of Contact, and *K*AF another recto-convexe angle under the very ſame ſides: and let *A*D be a right-line tangent upon the arch AF: therefore the recto-convexe angles of Contact D*A*F and *K*AB being equall, the right-lined angle *K*AD is the difference of the recto-convexe angle of Contact *K*AB and the other recto-convexe angle *K*AF under the divarication of the very ſame ſides. So as it is impoſſible to divide this, or any other angle whatſoever, which is not iſoclitical, to divide, I ſay, all of it into any numbers at pleaſure given of parts, which ſhall be homogeneal all of them one unto another: for how many ſoever be homogeneal, the angle of Contact, or that which is taken out of it, or that unto which it adhæres, will have and make heterogeneity.

And if you ſay, how can recto-convexe angles of Contact be ſaid to be parts of ſuch concrete and compoſite angles, if unable by any multiplicity to exhauſt the compoſite angle? omitting the anſwer that parts are ſometimes eſſential and of the definition, and yet by no multiplicity can equalize the whole; as four angles in the definition of a Tetragone, and a foot line in the definition of a foot Cube: I anſwer chiefly, that where the integral is heterogeneal as here, and made up, and properly and naturally reſoluble into ſeveral heterogeneal parts, and connot be divided into any numbers at pleaſure of parts all homogeneal, there ſome parts may never be able by any multiplicity to equalize the whole, or ſome other heterogeneal parts. And elſs-where that under the ſame coaptable ſides may be angles different in their ultimate kind in ſome farther reſpects, though not without proportionableneſs in this inſtance, is moſt apparent among right-lined angles, by comparing a right right-lined angle,

gle, and an acute right-lined angle, and an obtuſe right-lined angle together; which all receive their ſpecifick differences from the ſpecifick differences of their inclinations: in right right-lined angles the inclination being no more one way then the other, no more from the angle ſide, then to the angle ſide, *Sc.* perpendicular: in acute angles the inclination being to the angle ſide: and in obtuſe angles the inclination being more eſpecially from the angle ſide.

*A*nd yet though we defend the quantitativeneſs of recto-convexe angles of contact we are equally obliged to aſſert their improportionableneſs to right-lined angles: nor will there be any difficulty in anſwering that ſuggeſſion you caſt, though in anothers name; that a recto-convexe angle of Contact is in proportion leſs then a right lined angle, as being both homogeneal; and that by the multiplicity of the recto-convexe angle of Contact, an angle may be made equal to a right lined angle, or greater, only by changing its kind, *Sc.* into a right-lined angle: *Viz.* in the ſame manner, as an acute right-lined angle, being leſs then a right, or any obtuſe right-lined angle, by its multiplicity may with change of its ſort and kind become equal, or greater then a right, or any given obtuſe right-lined angle. To which may be anſwered, omitting what kind of diſtinction it is which is between acute, right and obtuſe right-lined angles, as not pertinent to the preſent controverſy; it would be well done to ſhew what multiplex of any recto-convexe angle of Contact, is equal to what right-lined angle, that ſo a right-lined angle might, upon that proportion, be formed equal to, or leſs then the recto-convexe angle of Contact; contrary to what has been clearly demonſtrated, and is generally by all conſented to in Geometry. Though acute and right right-lined angles are leſs then any obtuſe whatſoever that are right-lined; however they hold proportion one with another: but recto-convexe angles of Contact cannot be demonſtrated to hold any proportion with any right-lined angle, but clearly the contrary. And as by the divarications of the ſides of an acute right-lined angle is made a Geneſis of a right and infinite obtuſe right-lined angle: there cannot ſo by the divarication of the ſides of a recto-convexe angle of Contact be effected any Geneſis of right-lined angles, but only of recto-convexe mixt-lined angles, whoſe ſides concurre by way of ſection; and between which and the recto-convexe angle of Contact is no proportion, as there is between the divaricated acute right-lined angle and the other right-lined angles created from that divaricacation. The reaſon of which is clear, for that the recto-convexe angle of Contact being demonſtrated of it ſelf to be leſs then any right-lined angle whatſoever, by the divarication of the ſides of it, there are continually greater and greater right-lined angles added to it, ſo creating an improportionality between the one, and the other.

The compariſon made between Cyphers and angles of Contact, to draw the one, as well as the other into the notion of nullities, is unhappy enough: for ſingle and ſeveral Cyphers are not greater, or leſs in power one then another (though they may make other figures to be ſo) as the angles of Contact are, and may be made larger, or leſſer, and many of them one without or within another, contiguouſly and continuouſly conjoyned together, with enlargement, or diminution of their angularity; which is impoſſible in indiviſibles, and unappliable to, and unintelligible of, mere nullities.

That your Lemma is without exception, and without proof might have been admitted: *Viz.* That two quantities by the ordinate application, or motion of a line, or plane, increaſing, or decreaſing proportionably, whether by a proportionality in the ſame, or different powers, when the ordinate application, or motion attains the end, and bounding term of the one quantity, it at the ſame moment reacheth & hath attained the bounding term of the other, and when it hath paſſed the one it hath paſſed the other. But the objection againſt the quantitativeneſs of the recto-convexe angle of Contact, which you would hereupon found, hath no reaſon to expect the like allowance. You ſay, the right-lined right angle at the point of Contact contained under the right-line tangent, and the diameter of a Circle, equally with the circumference intercepted, increaſeth, or decreaſeth by the motion of the diameter upon the point of Contact, as a Center; which is true and acknowledged: and whereas you ſay, that therefore when the diameter leaves nothing at all of the circumference in its circumvolution about, un-run over, but attains the laſt bounder and termination of it, in the point of Contact, then as the circumference is quite exhauſted and vaniſhed, ſo is the angle too: this alſo we acknowledge to be undeniable. But whereas when the diameter is come ſo near the right-line tangent in its circumvolution upon the angular point of Contact, as to intercept nothing at all of the circumference between them, you then imagine ſtill an angle remaining, which you ſay, is either the recto-convexe angle of Contact, or not leſs then it; you herein forget the force of your own manifeſtly true lemma, which you took ſo much pains to prove; except againſt Geometrical,

cal demonſtration you could diſcover a poſſibility of dividing a recto-convexe angle of Contact by a right-line: for it is out of doubt and in Geometry, as above demonſtrated, that when the diameter in ſuch circumvolution intercepts nothing of the circumference between its ſelf and the right-line tangent, the diameter is then coincident, and the ſame right-line with the right-line tangent; and of the former angle therefore, hath left nothing at all, becauſe of the coincidency of the two right-lines, whoſe parts can in that poſiture have no inclination one unto another: and therefore there is not ſo much as the leaſt angle of Contact, or any other angle whatſoever, left after this circumvolution: ſo as the whole Argumentation is a long arrow out of a ſtrong Bow, but quite beſides the mark.

It is a ſeeming weighty objection that which is urged out of opticks, and the uſage in that ſcience, to demonſtrate in conical figures, the angles of incidence and reflexion to be equal, only with reſpect to the right-line tangent, touching the figure in the point of incidence and reflexion, without ſpecial reſpect to the curvature in the conical ſection. But hereto without wrong, either to truth, nature, or that noble ſcience, may be upon good grounds anſwered. 1. That opticks is not pure Geometry, and obſtructed ſtereometry, and mathematicks; wherein quantities, menſurations, and proportions are conſidered merely as in themſelves without relation to matter, and the uſes to which in other faculties they are applicable: but in opticks, is an improvement of what in nature may be obſerved about luminous and viſual beams, and luminous mediums and objects, by mathematical demonſtration and aſſiſtance. Now 2^{ly}. Nature doth not tye it ſelf in its wayes ſo ſtrictly to an indiviſible, abſolute, vertical punctilio, that if it cannot reach, without impediments, that mathematical exquiſitenes, it will not act at all. If heavy things on the clive ſide of an hill cannot deſcend in a direct line towards the Center, however they will ſeek it in the neareſt oblique line poſſibly they can, ſo long as the motion brings them nearer: all animals are not formed in the ſame mould, or with equal ſtrength of Body, or vigour of ſpirit. So as in theſe optical inſtances, the brave *A*uthors fell not in the leaſt ſhort of their task, when they had demonſtrated the moſt equal right-lined angles which nature could caſt it ſelf into, in the alledged caſes, *Viz.* angles of incidence and reflexion equal, if compared with a right-line tangent at their point. For as elſewhere, they are but very few points in moſt conical ſections, where, by a right-line, their angularity can be divided into equal parts: nature therefore when it cannot attain its prime and general deſign, which is a perfection and abſoluteneſs in its work, it doth not therefore ſlugge and do nothing, but ſeeks to approach its firſt deſign as near as poſſible: and that is in making with right-lined beams equal angles reſpectively to the right-line tangents at the points concerned; which angles ſo formed at thoſe points with reſpect unto the right-line tangents, are as is elſewhere ſhewn, either the leaſt of the greater right-lined angles, or the greateſt of the leſs right-lined angles: that in ſuch curvatures with great judgement the quality of the angles of incidence and reflexion, in beams paſſing by right-lines, as affected for their directneſs and ſhortneſs, and as near as poſſible endeavoured by nature, is in demonſtration referred to examination at the right-line tangents of the ſame points; an abſolute equality by right-lines to be made, being moſt what impoſſible, and that demonſtrably in ſuch curvatures: ſo as cauſeleſs was the exception which was made againſt the demonſtrations of the noble Perſpectiviſts: nor ſtood, either they, or nature in need of that improper, lame ſolution and help, by making recto-convexe angles of Contact to be neither angles, nor quantitative.

The truth of the angularity and quantitativeneſs of recto-convexe angles of Contact thus aſſerted; however, the reſt of the things in this controverſy cannot be determined without a clear underſtanding of what homogenealneſs it is that is requiſite for proportionals, and which is mentioned in the definition of proportion. To bring our ſelves unto the right underſtanding of which, know there are three wayes whereby one thing may be ſaid to be greater then another.

1ſt. Improportionably, and by the whole kind; as in heterogeneals: in which ſometime the leaſt of the one is ſtill, beyond all proportion, greater then the greateſt of the other: as the leaſt ſurface is greater then any line; for a longer line may be drawn in the leaſt ſurface, then any the longeſt line, that can poſſibly be given: and ſo the leaſt body is improportionably, and heterogeneally, and by the whole kind greater then any ſurface: for by the dividing, or altering the figuration of the leaſt body, it may be made, by its perimetry, to exceed the quantity of the greateſt ſurface, that poſſibly can be given: ſo a point, or a circle of a foot diameter, is ſaid to be leſs then the whole world; which is to be underſtood, as by the whole kind, and without proportionableneſs. And ſo I doubt not, but it will be confeſſed, that the leaſt recto-convexe angle of Contact, is greater then the greateſt acute, right or obtuſe right-lined angle: and the leaſt right-lined angle, then the greateſt recto-convexe angle of Contact.

2'y. One thing is greater then another infinitely, or, if you pleaſe, indefinitely: ſo an infinitely, or indefinitely inlarged line, is longer then any given line, without limitation, and ſo conſequenely without proportion. Or,

3'y. One thing is greater then another, rateably and according to the proportion, which they hold at the ſame ſtandard, whereby they are after the ſame manner meaſured, in the ſame kind in which they are compared. So one lenght, or line is longer then another, according to ſuch a proportion of length when both are meaſured as lengths at the ſame ſtandard upon an indefinite line; and this, whether the lengths, or lines be, one, both, all, neither, or none of them directly, or indirectly poſited, in right, or crooked lines; and whether the one be a depth, and the other a height, or another a breadth, or a fourth a periphery. So one number is greater then another number, according to ſuch a proportion: and, though it is well known to be determined in Philoſophy, that numbers are of different kinds, yet for the proportion they hold one to another in their common nature, there cannot be denied unto them the truth of a mathematical and analogical homogenealneſs.

And as things are ſaid to be ſeveral wayes one greater then another ; ſo homogeneity wants not its ſeveral acceptations. What homogeneity is , is not at all any where expreſſely defined in the mathematicks ; but we are left at large , rationally to collect , what is by that Term in thoſe faculties to be underſtood. The moſt uſual acceptation of the word homogeneity in Philoſophy is to compare any diviſible being with the parts into which it may be divided : thoſe things being ſaid to be homogeneal , which cannot be ſeparatingly divided into parts of any other name , or nature then the whole is : as the leaſt ſeparable part of water is ſaid to be water ; of wine , wine : and the leaſt ſeparable part of a line , a line : and thoſe things are ſaid heterogeneal , which by poſſibility may be ſeparatingly divided otherwiſe ; *Viz.* into parts of different name and nature from the whole. All which , as appears , hath its dependance upon the ſimilar , or diſſimilar natures of the whole and parts. So all ſolids , ſurfaces , lines , and plane angles may be ſaid to be homogeneal ; for by dividing and ſeparating them part from part , every part of the ſolid is a ſolid, of the ſurface a ſurface, of the line a line, and of the plane angle a plane angle: and by taking number in a large ſenſe , every part of a number may be ſaid to be a number. But then if you deſcend lower in numbers and angles, even and odde numbers, &c. and right-lined and mixed-lined and crooked-lined angles will ſcarce be able to defend their homogeneity at this touch: for Even numbers may be divided into parts that are odde: and a right-lined plane angle may be ſevered into parts whereof the one may be a recto-concave angle of a ſemi-Circle, the other a recto-convexe angle of Contact; of which laſt mentioned ſevered part you ſay, it is no angle, and therefore it muſt be heterogeneal: and we aſſert it heterogeneal, becauſe it is improportionable, and its quantity not menſurable after the ſame manner, that the quantity of right-lined angles is meaſured.

And with reference to the ſame acceptation, two diſtinct beings, being compared together, as to homogeneity and heterogeneity, they are then ſaid to be homogeneal, when both having the ſame name and nature, the one may be the ſevered part of the other, or both together the ſevered parts of an integral of the ſame both name and nature. So all ſolids are homogeneal: all ſurfaces: all lines: all plane angles and all numbers whatſoever.

But ſometimes alſo in an abuſive acceptation, the agreeing, or diſagreeing of things in ſome general, or ſpecial kind and nature, paſſeth for a certain kind of homogeneity or heterogeneity among them : ſo bodies and ſurfaces and lines in reſpect of their common agreeing in quantity, continuity, longitude, latitude and profundity, are partly homogeneal, partly heterogeneal : ſo numbers in reſpect of their general quantitativeneſs, parity, imparity, ſymmetry and a ſymmetry &c. Are partly homogeneal, partly heterogeneal; and ſo ſolid, ſphærical, and plane angles, and all their ſeveral kinds are partly homogeneal, and partly heterogeneal.

So as from any of the former acceptations it is no way poſſible to determine what is that mathematical homogeneity which is every where hinted unto us in the definitions of proportion ; for the bolting forth of which we ſhall be conſtrained of neceſſity to betake our ſelves to another courſe, in the queſt of which, it is not likely but we ſhall meet with ſome abſtruſeneſs and difficulty.

To make therefore an Eſſay; proportion being the habitude of the compared magnitudes according to quantity, it is an homogeneity in quantity which is only herein required; not an homogeneity in ſubſtance, quality, ſite, or other kinds and manners of being: for heterogeneity in any of thoſe other reſpects, as ſubſtance, quality, ſite, or other kinds of being, doth not hinder, but they may notwithſtanding be one proportionable to another, if any quantitative homogeneity be to be found amongſt them. So the heterogeneity of figure and figuration, which is between a Circle, pentagone, ſquare, triangle, ſurface of a pyramide, &c. Hinders not, but they may be compared and hold proportion in reſpect of their Area's and ſuperficial contents; wherein they retain a quantitative homogeneity: and thoſe of them which are meerly bounded planes, alſo have a homogeneity in reſpect of their perimetries, or the like quantities which they hold in common. So crookednes and rectitude are different kinds of poſiture, and make an heterogeneity in ſituation; yet a right-line and a crooked-line, hold ſtill a quantitative homogeneity, in reſpect of their longitude and extenſion. So the different poſitures of the ſurſaces in a ſphære, and in a cone, and in a plane, hindereth not their analogy in Area and quantity.

But further to purſue this diquiſition; though it be homogeneity in quantity, which is herein underſtood yet it is not the being quantities, or magnitudes in general that doth make up, or can compleat the homogeneity hinted in the definition of proportion: for then lines, and bodies, and numbers, and ſurfaces, being all of them magnitudes and quantities, there ſhould be a proportionablenes between lines and bodies, bodies and ſurfaces, ſurfaces and numbers; which in ſuch a general-homogeneity it were vanity to look for. In like manner, though plane, ſphærical and ſolid angles be all angles, yet I think there are very few, that becauſe of that their general homogeneity will expect a proportionablenes amongſt them.

Beſides as little neceſſity is there of reſtraining this homogeneity of magnitudes, unto their very laſt, and in every reſpect, ultimate kinds: except you will underſtand it to be in reſpect of ſome indefinite, moſt pure abſtracted quantity, in which the compared magnitudes are to be each of them meaſured. For though ſome referr all numbers unto quantity, diſtinguiſhing them into ſeveral kinds, not only from their parities and imparities, with all their variations thereupon, but making every number to be of a different kind, receiving its ſpecification from the laſt unite; as ſome alſo referr unto quantity all lines, which they diſtinguiſh into ſeveral kinds, & the like ſpecifick diſtinctions are made by others of ſurfaces and bodies: yet between proportionals, for their homogeneity, is not, we confeſſe, in this ſenſe required, that both magnitudes be of the very ſame ultimate kind: for hyperbolar, parabolar, elliptical, circular arches, and right-lines are homogeneal in length, extenſion, and as lines, ſo holding analogy, though in reſpect of rectitude and curvature, and their ſeveral kinds of curvature they be heterogeneal. So all numbers, even, odde, commenſurable, or incommenſurable, however, as above is ſaid, heterogeneal, yet as multitudes and numbers in general, and the accounts how oft an unite and its parts is poſited, or how far in account is proceeded, whether retrogradely or progreſſively from an unite; ſo, all numbers are homogeneal, holding mutual analogy one to another. So notwithſtanding the diſtinction between acute, right, and obtuſe, right-lined angles, all truth and exquiſitenes of proportion, as the meaſure of their quantitative relation, is moſt apparent amongſt them.

Which again doth further ſhew unto us what and how great is the difficulty of limiting the homogeneity expreſſed in the definition of proportion; ſo as neither to ſet it uſeleſly too high, nor ſink it lower then is neceſſary for proportionality. For right right-lined angles are proportionate, one to another, but there can be amongſt them no proportion of inequality: and no given acute right-lined angle can have ſo ſmall a proportion to any acute right-lined angle, as it may have and hath to a right right-lined angle, or to any obtuſe right-lined angle: and though all even numbers hold analogy and proportion one to another, yet even numbers are not capable of all diverſities of proportion; as in uneven numbers, duple and ſubduple proportions are impoſſible: nay numbers in general, I mean unbroken and integral numbers, are not capable of all kinds of analogy; as particularly not of ſuch proportions as are aſymmetral.

But not to be nice in my thoughts to my friend, I never in this point and queſtion underſtood any other thing to be meant by homogeneity in the definition of proportion, then a menſurablenes of the quantities of two, or more magnitudes in the ſame indefinite quantity for kind, as the meaſure of their quantity and of the quantity of all their homogeneals, the menſuration therein ſtill being according to that ſame kind of quantity of which the indefinite quantity is. And ſo proportion is the rate and habitude, which the rateable magnitudes hold mutually one to the other in reſpect of the ſame way of meaſuring their quantities, or in reſpect of the ſame kind of indefinite quantity in which their quantities are meaſured. And upon this gloſs as the true and genuine meaning of this mathematical homogeneity, I ever underſtood that poſtulate to be founded in which is required and granted, ſo to multiply any given quantity, as to exceed any other given quantity whatſoever of the ſame kind. For if that menſurablenes in the ſame indefinite quantity, as a meaſure, and according to that ſame kind of quantity of which the indefinite meaſure is, were not the very thing deſigned by mathematical homogeneity, the matter of the Poſtulat were not fit to be granted without proof: for it is becauſe they are meaſured in the ſame indefinite quantity for kind, and according to the ſame kind of quantity, *i. e.* they have the very ſame way of meaſuring in the ſame indefinite quantity (which is their homogeneity, by neceſſity of conſequence creating a proportionablenes between them) that the leſs by multiplying may be made greater then the greater, and the greater by a continual cutting off ſtill more then half may be made leſs then the leſs.

And though hereby homogeneity and proportionablenes be not made to be the very ſame thing, however in the mathematicks, where the phyſical natures of things are not inquired into, the one by a neceſſary conſequence doth immediately flow from and is annexed unto the other; and becauſe of their neceſſary connexion, in uſual ſpeech and acceptation, the one may be allowed to be taken for the other. And when in the definition of proportion, proportion is ſaid to be the mutual habitude of magnitudes of the ſame kind according to quantity, or if you pleaſe multiplicity; the meaning is no other, but that proportion is the mutual habitude of magnitudes, which have their menſuration after the ſame manner, according to quantity, or multiplicity; taking the word multiplicity in a large ſenſe: *i. e.* according to the quantity and multiplicity, which they have each to other in the ſame indefinite quantity and meaſure, upon which they are in the ſame manner and according to the ſame kind of quantity meaſured. However in natural Philoſophy for very weighty reaſons homogeneity and proportionablenes are to be acknowledged of very diſtant and different natures. So I preſume in Mathematicks it would be taken for a ſolæciſme to ſay a body and a line were homogeneals and of the ſame kind, becauſe all ſeparable parts of each agree in their being all of them continuous quantities; though in the Phyſiological ſchool that they do concenter and meet in the ſame general nature is not deniable; and ſo they may carry a ſeeming ſhadow of homogeneity, ſo far as homogeneity may be abuſively wreſted to denote any ſuch common agreement in a general notion and nature. So it would be a ſolæciſme in Mathematicks to ſay, that a ſolid angle, a ſphærical angle and a plane angle were all homogeneal, becauſe they are all angles and every ſeparable part of each is an angle: but to prove Mathematical homogeneity, the menſurablenes of the quantity of the compared magnitudes in the ſame indefinite quantity, or meaſure for kind, and according to the ſame kind of quantity with the indefinite meaſure is to be made out: for that all are quantities, or all angles, makes them not in the mathematical ſchool homogeneal; except by reaſon of this menſurability of the quantity of both in the ſame indefinite quantity, or meaſure, according to the ſame kind of quantity, the leſs by multiplying can be made greater then the greater, and the greater by dividing leſs then the leſs. And indeed this is the true homogeneity, not denoting a general conveniency in their natures in reſpect of ſome abſtracted notion, but rather a ſpecial identity by reaſon of their menſurablenes in the ſame ſubſtrate kind of quantity and meaſure, only with difference, or proportionablenes of magnitude between both the wholes, and all the leaſt, or greateſt, proper, *i. e.* homogeneal parts of each: as if one be a line, ſo is the other, and the greateſt and leaſt proper, *i. e.* homogeneal or homometral parts of each are lines and proportionable to either. For, if beſides the menſurablenes of the quantity of the compared magnitudes in the ſame indefinite quantity, or meaſure, be not alſo added that condition, that in that indefinite quantity, or meaſure they are alſo menſurable according to the ſame quantity for kind, of which the indefinite meaſure is, and ſo conſequently proportionable one to another;

by nothing will it yet be determinable, whether all angles be not homogeneal: for in every angle, though of ſeveral kinds, every part of each angle is an angle: nor will it be determinable, whether all continuous quantities be not homogeneal; for every part even of heterogeneal continuous quantities is a continuous quantity: nor will it be determinable whether all numbers be not homogeneal; for that all numbers are of the ſame kind will be found a doctrine of very hard digeſtion any where ſave in the mathematick ſchool: yet according to this explication of mathematical homogeneity, notwithſtanding the diverſity of their kinds in other philoſophical conſiderations, they have in them a clear mathematical homogeneity: and even an unite, which in other parts of Philoſophy is not paſſable for a number, will fall alſo within the verge of the ſame homogeneity; as will alſo all the parts of an unite, whether commenſurable, or incommenſurable. And this explication of mathematical homogeneity will be allowed its due right and juſtification more eaſily, by thoſe who note how the main matter and deſign of mathematical definitions is but exegetical to clear up what is meant by the terms in thoſe ſciences uſed: for what other occaſion could there be in the Mathematicks to intermeddle with homogeneity, but to explain the noble points of proportion and proportionality.

And yet though in mathematicks there be ſuch a grand affinity between the proportionablenes and homogeneity of magnitudes and in common uſe and ſpeech the one may be put for the other , yet as above the notions are eaſily diſtinguiſhable by the underſtanding: *Viz.* two , or more magnitudes are ſaid homogeneal chiefly in reſpect of the ſame way of meaſuring them , or in reſpect of the ſame kind of indefinite quantity in which they are both menſurable ; but they are ſaid proportionable in reſpect of the mutual habitude and quantitative relation which is between themſelves upon ſuch their menſuration , in the ſame way, or according to the ſame kind of quantity. So all finite lines are homogeneal , as menſurable in the ſame indefinite line ; but that one finite line is double to another is the habitude of the one to the other declared upon that menſuration.

That upon the whole matter mathematical homogeneity doth not inſiſt only upon identity in kind at large , or reſtrain unto identity in kind at all points , and in every reſpect and conſideration ; but it is identity in quantity and therein particularly in the manner of the menſuration of their quantities , in which the homogeneity of magnitudes is chiefly lodged: and that is that which I ever underſtood in thoſe words (according to quantity, or if you pleaſe, rather quotuplicity) which are in the definition of proportion, *viz.* quantity and quotuplicity in the ſame way of meaſuring.

And if you urge here, according to this account I muſt conclude all incommenſurables to be heterogeneal; I anſwer, not in the leaſt: for though they may have no common meaſure, which can by poſſibility exactly meaſure both or all, yet there may be a common indefinite meaſure, in which each may exactly meaſure forth its own quantity. As for example in fig. 24. Let *A* and B be incommenſurable lines, and DC. an indefinite line, beginning at D, and on the part of C infinitely produced: from the point D in the line DC take the line DE equal to the line A: and alſo in the line DC take the line DF equal to the line B. here in the line DC as a common indefinite meaſure of their homogeneity, the two lines A and B, though incommenſurable, have meaſured forth themſelves by the lines DE and DF. So in fig. 1. upon A the angular point of the right-lined angle BAC, as Center, draw the Arch dfe cutting the line AB in the point d, and the line AC in the point f, and the right-line Ae in the point e, ſo as the Arch df be incommenſurable to the

Arch fe. Here the two right-lined angles dAf and fAe are incommenſurable, yet have a common way of meaſuring their quantity and proportion, *Viz.* by Arches of Circles drawn upon the angular point, as center, intercepted reſpectively between the ſides: as by the *Arch* dfe, the one being in that *Arch* meaſured by the Arch, df, the other by the Arch fe. So as by mathematical homogeneity is underſtood an homometricalnes, or autometricalnes with the neceſſary conſequent of a rateablenes therein, without any neceſſity of ſymmetricalnes between themſelves at all. For ſtill, if in homogeneity beſides homometricalnes in the ſame indefinite quantity be not alſo included, that the menſuration of both be according to the ſame kind of quantity, of which the indefinite quantity, in which they are meaſured, is; a point and a line may meaſure themſelves in an indefinite line, and a line and a ſurface may meaſure themſelves in an indefinite ſurface, and a ſurface and a ſolid may meaſure themſelves in an indefinite ſolid. But true mathematical homogeneity is when two, or more quantities being menſurable in the ſame indefinite quantity, or meaſure, and according to the kind of the indefinite quantity are by conſequence rateably, *i. e.* proportionably and homometrically equal, or the one bigger then the other, not the one infinitely bigger than the other, as an infinite line is longer then a finite line, for between ſuch it is acknowledged there is no proportion nor mathematical homogeneity; nor the one bigger then the other by the whole kind, as every ſolid is bigger then any ſurface, and every right-lined angle then any recto-convexe angle of Contact: and proportion is the rate, or quantity of their mutual habitude in that their homogeneity, or homometricalnes: *i. e.* proportion is the rate, quantity, or account of their proportionality. And applying our minds rightly to conceive of homogeneity as mathematical; of neceſſity ſuch, and no other can be the notion of it, thereby making two quantities to be rateably, *i. e.* proportionably equal, or the one bigger then the other, and according to their capacity and poſſibility in any rate and proportion ſo to be conſtituted and ſet out. For the genuine well known notion of homogeneity in general, what is it but that all and each of the proper, *i. e.* homogeneal parts, and the whole, fall under the ſame Denomination and nature, as if one be a line, all the reſt to be lines, if one be water, or ſtone, or oyle, &c. all the reſt of the parts and the whole to be ſo alſo? And in the mathematicks what is ſaid properly and homogeneally to be a part of any magnitude, but only ſuch leſſer magnitudes ſeparable from it as are able to meaſure out, *i.e.*by their multiple to exhauſt the firſt magnitude? For a ſurface is not ſaid to be an homogeneally proper part of a ſolid: nor a line of a ſurface. And by ſuch meaſuring forth of the integral magnitude by its homogeneally proper, *i.e.* exhauſting parts, is not underſtood the ſymmetry of the parts, and whole, for the whole and its true homogeneal parts, may be incommenſurable; but that which is underſtood is the true mathematical homogeneity of the whole and the parts in reſpect of their common way of meaſuring, and the proportionality, which is thereupon lodged between them So as by laying theſe undeniable remarks together, mathematical homogeneity in reſpect of the ſame integral magnitude, is that every part thereof being proper, *i.e.* menſurable in the ſame indefinite meaſure according to the ſpecial kind of its quantity, and therefore able to exhauſt the whole, be all of them only in reſpect of their quantitativenes, not their figuration, or other reſpects, of the ſame nature and denomination, and any proportion whatſoever (according to the capacity of ſuch kind of quantities) poſſible to be conſtituted amongſt them. And by conſequence ſeveral magnitudes are then ſaid to be mathematically homogeneal, when being the one able to exhauſt the other, & ſo in proportion one unto another, both and all the proper parts of each, all of them, as being of the ſame nature fal under the ſame quantitative denomination.

And here it is worth our noting, how ſom magnitudes are ſo homogeneal, that they cannot be ſeparatingly divided into parts, or magnitudes which are heterogeneal; but all the parts into which they are ſeparatingly divided will ſtill be homogeneal, both one with another and with the whole: So a line can no way be ſeparatingly divided into parts which are magnitudes, but each is a line homogeneal to the whole and to all parts whatſoever of whatſoever other line: neither can a body and ſurface be ſeparatingly divided, but into bodies and ſurfaces; each of whoſe parts are ſtill able to exhauſt the whole and are thereunto proportionable. But ſuch an abſolute homogeneity is not in all other magnitudes, eſpecially in heterogeneally concrete magnitudes, but that the whole may be divideable into parts ſeparable one from another, which yet are heterogeneal: as in the preſent caſe of angles a right right-lined angle is divideable into a recto-convexe angle of Contact, and the recto-concave angle of the ſemi Circle; which are ſeparable one from another, and therefore truly parts; yet, neither of them is homogeneal to the whole, nor one of them unto the other: for neither can the whole right right-lined angle, nor the recto-concave angle of the ſemi Circle ever be exhauſted by any number whatſoever of ſuch heterogeneal parts, as is the recto-convexe angle of Contact: nor ever any equality, or other proportion can poſſibly be ſhewn between the right right-lined angle, and the recto-concave angle of the ſemi Circle, becauſe there is no way poſſible in which their quantities can be proportionably menſurable. For not without very good reaſon unto all magnitudes are to be allowed their ſpecial properties; as to all poſitures and figuration, theirs. To angles theſe things are peculiar, being otherwiſe in other magnitudes: *Viz.* in angles which are truly and on all hands confeſſedly homogeneal, you cannot to any given angle, ſet forth another of the ſame kind in any given proportion at pleaſure: for every right-lined angle by a neceſſity of nature, muſt be leſs then two right right-lined angles: and in a plane all the angularity at any point cannot exceed what the circumjacent ſpace, or plane is capable of; which is only four right right-lined angles. That as number cannot be infinitely divided without fraction; ſo angularity cannot at pleaſure at the ſame point, in the ſame plane, be inlarged: whereas ſome other quantities have both infinite diviſibility and infinite multiplicability.

So another property of the magnitude of angles is, that it may not only in notion and ſpeculation, but in truth and ſeveringly, be divided into parts either able, or unable to exhauſt the whole: as when a right right-lined angle is divided into the recto-concave angle of a ſemi Circle, and a recto-convexe angle of Contact: you may ſever them the one from the other: and angularity is equally, if not much more apparent in the recto-convexe angle of Contact, then in the recto-concave angle of the ſemi Circle; yet the one of them is demonſtrated and confeſſed unable ever to exhauſt the right right-lined angle, the other not.

A further property of the magnitude of angles is, that ſometimes the ſame part, which hath already been ſevered from it, cannot exactly and immediately again by its equal be ſevered from it on the ſame ſide, though the remaining angle be by the whole kind greater. So after a recto-convexe angle of Contact is taken out of a right right-lined angle, there cannot again immediately on the ſame ſide, be ſevered from the remaining angle another angle equal to the recto-convexe angle of Contact, which was before ſevered from it. If it can, let it be performed.

Alſo the diviſibility which is in the magnitude of all Angles, though boundles and infinite, in ſome however, leaves the dividing of the *A*ngle into two equal parts, impoſſible: as notwithſtanding the perpetual diviſibility of lines, the ſide and diameter of a ſquare are left incommenſurable. So ſome other angles may be divided into two equal parts, but it is impoſſible to divide them into three equal parts: as convexo-convexe angles of Contact, with infinite other convexo-convexe angles and concavo-concave angles being contained under equal, uniform and anſwerable Arches.

To conſigne this point, the principal thing we have laboured herein to dilucidate, & as we doubt not have effected is, that mathematical homogenealneſs is not an homogeneity of all the parts whatſoever, that are in the magnitudes, which are homogeneal in reſpect of ſome ſpecial way of meaſuring their quantities; or an undivideableneſs of ſuch homogeneal magnitudes into parts otherwiſe heterogeneal according to which acceptation the word is chiefly taken in other parts of Philoſophy: for there is no right-lined angle whatſoever, nor any other angle whatſoever, but as is up and down herein ſhewn may be ſeparatingly divided into heterogeneal parts: but mathematical homogeneity is homogeneity in the way of meaſuring the quantity of the compared magnitudes, *Sc.* in the ſame indefinite meaſure and quantity, and according to the kind of the indefinite meaſure, and which thereupon follows, a proportionality between them, in reſpect of their common way of meaſuring: and of this mathematical homogeneity, fair footſteppings are to be found every where in the deducing of thoſe demonſtrations which concern proportions and proportionals. That, ſuch magnitudes as have no common way of meaſuring their quantity, as weights and meaſures, are heterogeneal: or if they have a common way of meaſuring in which they may meaſure themſelves, but therein do not meaſure themſelves according to the ſame kind of quantity with the indefinite meaſure, and ſo want proportionality, yet notwithſtanding they are heterogeneal: as all recto-convexe angles of Contact, all recto-concave angles of ſemi Circles, all recto-convexe angles of ſemi Circles, all acute, or right right-lined angles, theſe may all meaſure themſelves, and in what order their ſides fall within, or without in any obtuſe right-lined angle whatſoever: yet becauſe this their homometry is only of the ſituation, or order, in which the ſides part from the angular point, but not of their quantity in an indefinite meaſure and according to the denomination of the ſame quantitative meaſure, ſo as to lodge a proportionality between the magnitudes ſo compared together in their common way of meaſuring; they are not, nor can thereby be vindicated from their otherwiſe innate mathematical heterogeneity: which concerning ſome of them is confeſſed on all hands and is without the verge of the controverſy. And as follows, angles are of a concrete nature, having in them ſomething quantitative and ſomething not quantitative: whereas that which is to be the indefinite meaſure of homogeneal quantities is to be conſidered abſtractly as quantity without heterogeneal concretion: ſo it is the circumference of a Circle that meaſures all right-lined angles. And when all plane angles are ſaid to be homogeneal, it is not in reſpect of a common indefinite quantity by which they are all meaſured, which the recto-convexe angles of Contact doe ſufficiently evince, but as is manifeſt it is only becauſe of the poſition and ſituation of the ſides in the ſame plane; which homogeneity is of no concern unto quantity, nor by any neceſſity can thereupon infer the conſequent of proportionableneſs.

But to proceed; as is ſaid, beſides the former mathematical and quantitative homogeneity and heterogeneity there is alſo an extramathematical and extra-quantitative homogenealnes and heterogenealnes in angles every where obſervable in their ſhapes, figures, poſiture of their ſides, ſuch like ſchematiſmes and other reſpects. In general as is above hinted, every part of a plane angle is a plane angle, even the recto-convexe angle of Contact, however you deny it to be an angle and quantitative: but then this is not a mathematical homogeneity, but only in reſpect of a certain figuration, in reſpect of the poſiture and ſituation of the ſurface in which thoſe angles are; ſhewing how all plane angles from the greateſt to the leaſt, agree in that particular of their general figuration, *Viz.* of having their containing ſides to lye ſtill in the ſame plane; whereby they diſtinguiſh themſelves from all other ſuperficial angles, which are heterepipedal, whoſe containing lines, or ſides lye in ſeveral planes: ſuch as are all ſorts of ſphærical, cylindrical and conical ſurface-angles. But if ever a mathematical and quantitative homogeneity be proved among all plane angles, you that know that it is not my uſe to ſtart from my word, ſhall hereby reſt aſſured, upon the firſt ſummons I will give up this cauſe.

And we are not to think ſtrange, that a figuration is aſſerted to be in angles; for if we ſeriouſly conſider, weeſhall find there is ſhape and figure in angles, as well as quantity; as lines, and ſurfaces, and bodies have their figurations, the poſiture of their parts, their ſhapes and forms, as well as their quantities and magnitudes: in each, their figuration being manifeſt; *Viz.* in lines, in reſpect of their lineary poſiture : in ſurfaces, in reſpect of their ſuperficial poſiture : in bodies in reſpect of their ſolid poſiture : and in the caſting of each of their ſchematiſmes quantity is involved, as length, breadth, depth, *Viz.* their quantities, and the quantities which are compounded of ſeveral, or all of them together.

And here by the figuration which we aſſert in angles, we cannot be thought to mean that any right-lined figure can be compleated, perfectly to bound up a plane on all ſides, by one angle; it being beyond the power of two: and three, being the leaſt number of angles, requiſite ſo to conſtitute and perfectly limite out a right-lined plane figure. And though ſome plane figures are perfected, and perfectly bounded, without any ſuch angles as are contained by ſides concurring by way either of ſection, or Contact, as namly all Circles and Ellipſes; yet the angularity of curve coincidence is every where found, or at pleaſure aſſignable in the bounders of ſuch figures. But our meaning is, a plane angle, though moſt what it do not by the continuation and production of its ſides, perfectly bound in and limit out a certain plane and ſpace on every ſide, however being the mutual habitude of concurring lines, it gives an imperfect figuration to the plane and ſpace on its part. And as a bounded plane cannot be without ſome kind of plane figure, ſo a limited angle ever implies in it an imperfect figuration of ſome ſort, or other. For figuration is the conſectary of material finiteneſs and limitation in the poſition of lengths, breadths, depths, ſurfaces and ſolidities; that every angle having its limits and bounds cannot be thereof deſtitute. And if the name of figures be ſo frequently given to hyperbola's, parabola's, and the like, which neither do, nor ever can by any poſſible production perfectly bound in their planes; what reaſon is there then why angles ſhould be denied an imperfect intereſt in the name. Beſides as a plane in its own general nature at large doth not denote any ſpecial plane figure; but the riſe of figures, I mean plane figures, is from the bounding of the plane: ſo it is in angles, as they by the mutual habitude of their concurring ſides give imperfect limits and bounds unto the ſpace and plane, ſo they therein make an imperfect figuration. That in angles ſomething of form and figure is to be noted as well as magnitude. And one line cannot concurre with and be inclined upon another, but an imperfect figuration will ariſe from that their mutual inclination. And the ſame two angles may have the inclination, *i. e.* the receſſe of their reſpective ſides one from another equal, though there be no analogy between the figurations of the angles, or the ſhapes in the which the ſides are inclined in the one and in the other. For by reaſon that in angles form and figure are to be obſerved as well as quantity; crooked-lined and right-lined angles may be equal in ſome particular quantity, yet other-wiſe not of the ſame kind: they having equality in ſome magnitude, but being diſtinct in the manner of their forming, figuration and conſtitution: as equality may be between a ſquare and a triangle, though figures altogether different in kind.

And in reſpect of ſuch their figurations, plane angles receive diſtinction, either from the diverſe manners in which their containing ſides do concurre, or elſe from the diverſe natures, and figurations of the lines under which they are contained, or, which is tantamount from the diverſity of the inclinations and inflexions, or rather inclinableneſſes and imflexibleneſſes by which they are inclined each to other, or from ſeveral of theſe grounds of diſtinction taken together.

Plane angles from the different manner of their ſides concurring, may aptly be thus diſtinguiſhed, *Viz.* into angles, whoſe ſides concurre by way of ſection; or elſe that have their ſides concurring only by way of touch in ſome ſingle ſingular point without mutual ſection; or elſe their concurrence is in curvature, where, after the meeting of their ſides in the angular point, the ſides do not in their productions depart one from another, neither by way of touch, nor ſection; but become, the production of the one ſide coincident with the other ſide; ſo as this kind of angles may aptly be called angles of coincidency, or angles of curvature: and in theſe lies the genuine Ratio and true account of the curvature.

From the diverſe figuration of the lines, under which a plane angle is contained, very many differences of angles may ariſe, according to the various diſtinctions, of which lines themſelves are capable, I mean ſuch lines, as fall not without the capacity and comprehenſivenes of the ſame plane: as that ſome are helicoidal, ſome parabolar, ſome elliptical, &c. But as of lines, ſo hence of angles, the chief and primary diſtinctions are eſpecially theſe; *Viz.* that plane angles are either right-lined angles, contained under two inclined right-lines; or not right-lined angles.

Not right-lined angles, are either mixed lined angles; contained under one right-line, and one crooked-line: or crooked-lined angles contained under two crooked-lined ſides. And from the ſeveral kinds of ſpecial, or ordinary curvatures, as Circular, elliptical, hyperbolar, &c. The mixt-lined, and crooked-lined angles, are capable of many farther and more particular diſtinctions ; but eſpecially from the ſite of the convexeneſs, or concaveneſs of the lines to , or from the angle ſide: though all ſuch ſecondary diſtinctions riſing from theſe two laſt mentioned heads , are as properly and pertinently referable to the other ground of diſtinguiſhing plane angles, taken from the differences which may be in the inclinations of the one containing ſide to the other. For a vaſt difference is in the inclination of a crooked-line , by obverting the concave , or convexe ſide to any other line. So the conſtituting a circular , or elliptical arch &c. For one ſide , makes a vaſt difference in the inclinations , becauſe of the difference in their curvatures.

Alſo another principal diſtinction of angles from their ſides, may be into angles, whoſe ſides are coaptable ; and by poſſibility may be coincident one with another : or elſe ſuch as have between them no poſſibility of coaptation and coincidence. Of the former ſort are all right-lined angles , and all concavo-convexe angles contained by *A*rches of equal homogeneal, uniform, regular , or anſwering curvatures: of the later ſort are all other ; whether mixt lined angles , or crooked-lined angles ; whether they be mixed crooked-lined angles , or unmixt crooked-lined angles. And conſequently thereupon , beſides the numerous diſtinctions of angles in reſpect of their different inclinations; ſuch as above mentioned; one is more eminently material above the reſt, that the inclination of the ſides, is ſometimes with an equability all along their production; though imagined to be infinitely extended, in ſuch lines as by poſſibility may with reaſon be imagined ſo to be: and ſometimes there is nothing of equability to be found in the inclination of the ſeveral parts of each ſide to the other; though it may be one of the ſides be a right-line, or an Arch of moſt equal, uniform, regular and homogeneal curvature. And this equability and inequability of the inclination of the ſides, ſtrangely alters the properties of angles. As in right-lined angles, for the equability of the inclination of the ſides, no parts of the one ſide are more inclined then the reſt unto the other ſide: and ſo in concavo-convexe angles of equal curvatures, no parts of the one *Arch* are more inclined then the reſt unto the other; but the one Arch is all along inclined unto the other, as at the angular point; and the inclination, which the one bears unto the other at the angular point, is obviouſly expreſſable as to the quantity of the receſſes, which they make one from another, by the inclination of a right-line to a right-line, except when the inclination of the Arches is equal to, or greater then two right right-lined an-

angles. And in ſuch crooked-lined angles, whoſe ſides have equability of inclination, the points, which from the angular point are at equal diſtances along the *Arches*, are alſo abſolutely at equal diſtance from the angular point, along the chords: and right-lined tangents at any two ſuch homologal points, where ever taken, alwayes meet and contain a right-lined angle, equal as to the quantities of the receſſes of the ſides, to the crooked-lined angle contained by the two Arches; as is obvious to demonſtrate, eſpecially in circular Arches. *A*nd the right-lined angle contained under the two right-lined tangents touching at the two homologal and anſwering points, which is equal to the iſoclitical crooked-lined angle, if the two right-lined tangents occurre on that ſide of the right-line connecting the homologal points on which the iſoclitical angle falls, then it is the angle contained by the two right-lined tangents into whoſe ſpace part of the ſpace comprized between the two *Arches* at firſt falls, which is equal to the crooked-lined iſoclitical angle: but if they occurre on the other ſide of the right-line connecting the two homologal points, *i. e.* averſely from the crooked-line iſoclitical angle, then it is the complement of ſuch an angle, which is equal to the crooked-line iſoclitical angle: but if the two right-lined tangents occurre in one of the homologal points, the angles either way contained under the two right line tangents are equal, *viz.* right right-lined angles; either of them making forth what is herein aſſerted. As in fig: 19. under the two circular iſoclitical arches bda and acn let there be conſtituted the iſoclitical angle b a c; and let the right-line a g touch the Arch a c n in the point a: and let the right-line af touch the Arch adb in the point a: ſo making the right-lined angle fag equal to the iſoclitical concavo-convexe angle b a c. Then take in the *A*rch a d b any point at pleaſure, *Viz.* the point d: and draw the chord a d. Then in the Arch a c n take the *A*rch a c ſubtended by the chord a c equal to the chord a d. Therefore becauſe of the iſocliticalneſs of the circular Arches the two points d and c are two homologal, *i. e.* anſwering points the one in the one *A*rch, the other in the other, *Viz.* the point d in the Arch adb and the point c in the Arch a c n. Then draw the right-line d c connecting the two homologal points d and c. *A*lſo draw the right-line d e touching the arch a d b in the point d; and the right-line c e touching the *A*rch a c n in the point c. *A*nd let d e and c e the two right-line tangents be produced till they meet in the point e; which in this figure is on that ſide of the right-line d c on which the concavo-convexe angle c a b lyeth. I ſay therefore

fore that the right-lined angle d e c contained under the two right-lined tangents d e and c e touching the Arches reſpectively at the homological points d and c is equal to the right-lined angle f a g, contained under the two right-line tangents f a and g a touching the Arches reſpectively at a the angular point of the iſoclitical concavo-convexe, angle. For the right-lined tangent f a cutting d e the other right-lined tangent of the ſame Arch a d b in the point h; and the right-lined tangent d e of the Arch a d b cutting the chord a c in the point k; upon this conſtruction the right-line d a is equal to the right-line a c; and the right-line tangent d h is equal to the right-line tangent h a: therefore the right-lined angle a d h is equal to the right-lined angle d a h and ſo to the right-lined angles a c e and c a g ſeverally. And therefore the right-lined angle a h e being equal to the two right-lined angles h d a and d a h taken together, and the right-lined angle h d a being equal to the right-lined angle c a g; the right-lined angle a h e is equal to the two right-lined angles c a g and d a h taken together. Therefore that which maketh each equal to two right right-lined angles; the two right-lined angles h k a and h a k taken together are equal to the two right-lined angles h a k and d a l taken together. Therefore the right-lined angle hka is equal to the right-lined angle dal. Therefore the right-lined angle cke is equal to the right-lined angle dal. And therefore that which makes either equal to two right right-lined angles, the two right-lined angles kce and kec together taken are equal to the right-lined angle dag, which is equal to the two right-lined angles dac and cag taken together: and the right-lined angle cag is equal to the right-lined angle kce: therefore the right-lined angle kec is equal to the right-lined angle dac: therefore becauſe the right-lined angles dah and cag are equal; alſo the right-lined angle kec &c. dec is equal to the right-lined angle hag &c. fag, which was to be demonſtrated.

But if the two right-lined tangents de and ce as in fig. 20. do not occurre towards the concavo-convexe angle bac, but on the other ſide of the right-line dc in the point e; then is the right lined angle dec contained under the two right line tangents de and ce touching at the homologal points d and c, not equal to the concavo-convexe iſoclitical angle bac, or the right lined angle, equal unto it, fag, but to its complement unto two right right-lined angles, *Viz.* unto the right lined angle fal; the right line la being the production of the right line ga. For as before by conſtruction, the right line chords da and ca to the homologal points d and c are equal:and the right line f a cutting the right line d e produced in the point h, the right lines d h and a h being two right lines tangents of the ſame Circle a d b, occurring, are equal. And let the right line a c produced, occurre with the right line d e produced, in the point k. As appears;the right lined angles a d h, d a h and c a g, as before, are equal; and the right lined angle a h e is equal to two right right lined angles, all but the two right lined angles h d a and h a d that is, all but the two right lined angles c a g and h a d. Therefore the right lined angle a h e being equal to the two right lined angles hka and k a h; the two right lined angles h k a and k a h are equal to two right right lined angles all but the two right lined angles c a g and h a d. Therefore two right right-lined angles are equal to the four right lined angles c a g and h a d and h k a and k a h. Therefore out of equals taking equals, the right lined angle h k a, which is the right lined angle c k e is equal to the right lined angle d a l. Therefore what on either ſide remains to make up two right right lined angles on either part; the two right lined angles k c e and k e c are together equal to the right lined angle d a g, which is equal to the two right lined angles d a c and c a g taken together. And producing the right line c c till it cut the right line a g in the point g; the right lined angles c a g and a c g and k c e are equal. Therefore the right lined angle k e c is equal to the right lined angle d a c. And becauſe the right-lined angle d a h and c a g are equal, therefore adding the common angle f a c, the two right lined angles d a c and h a g are equal. And therefore the right lined angle k e c is equal to the right lined angle h a g. And therefore their complements unto two right right lined angles, *Viz.* the two right lined angles d e c and h a l are equal; which was to be demonſtrated.

So in fig. 22. if the right line tangent de paſſe directly unto the other homologal point c, as it doth when the iſoclitical concavo-convexe angle is equal to a right right lined angle, and the homologal points d and c are taken at quadrantal or other diſtances from the angular point; then moſt manifeſtly the right-lined angles d c g and f a g are equal, being under the two and two reſpective right lined tangents; and between the angles and their complements unto two right right lined angles is no difference, as appears by what is in the former demonſtrations. So in fig. 23. if the homologal points d and c be ſo taken, that the chords d a and c a be the diameters: then producing the right line tangent c e till it occurre with the other right line tangent d e in the point e, and with the other right-line tangent f a in the point f; the right lined angles c f a and f a g are equal. And in the trapezium d e f a the two right lined angles e d a and d a f are each of them a right right-lined angle, therefore the right-lined angle d e c is the complement of the right lined angle e f a unto two right right-lined angles, *i. e.* it is the complement of the angle f a g unto two right right lined angles; which was to be demonſtrated. And like demonſtrations may be formed upon every other caſe. But where the iſoclitical arches are not circular the demonſtrations muſt vary according to the propriety of every ſeveral curvature, notwithſtanding its equability and iſocliticalneſs.

But to return, in angles contained by ſides whoſe inclination each to other is without any ſuch equability, (except only in unmixt crooked-lined aniſoclitical angles) the account of the homologal points along the chords and along the arches is ſtill different: but in none of them do the right line tangents from the two and two homologal points, ſtill in their meeting make the two ſame angles which are made by the two right lined tangents at the angular point; nor can any right line angle expreſſe the inclination, which the ſides have each to other at the angular point.

Hence is manifeſt how the ſame angle may from ſeveral of the grounds of diſtinction here propoſed be referable to ſeveral heads, or kinds. So right lined angles as iſoclitical, have alwayes equability of inclination; and the concurrence of their ſides is alwayes by way of ſection and cannot be by Contact, or coincidency. So mixed lined angles being aniſoclitical have always inequability in the inclination of their ſides; and their concurrence may be either by Contact, or Section, but never by coincidence. And in crooked lined angles, their ſides may have either equability, or inequability of inclination: and accordingly the concurrence of the ſides may be by ſection, or Contact; and with, or without poſſibility of coincidence Sc. Iſoclitical crooked-lines, but they muſt be poſited convexo-convexely, or aniſoclitical crooked-lines poſited whether convexo-convexely, or concavo-convexely, or a right-line and a crooked line, any two of theſe may touch without cutting, and ſo the angular ſides have inclination tangent and not ſecant, So iſoclitical concavo-convexe angles may have ſides circumducted to coincidence: but the ſame ſides poſited concavo-concavely, or convexo-convexely become aniſoclitical in the circumduction, one in reſpect of the other; yet either is iſoclitical ſometimes and in ſome caſes with the production of the other.

From theſe things though diſtinguiſhing angles into their ſeveral kinds, only with reſpect to the diverſity of their figuration, may however more abundantly appear how unmanageable a task they take upon themſelves, who to exclude recto-convexe angles of Contact from being angles, and from quantitaveneſs, would force all plane angles to be of the ſame kind, allowing no ſpecifick difference poſſible among them. Not here to purſue what other diverſities in kind may be obſerved among angles; how can the inclination of a crooked-line upon a right-line differ leſs then in kind from the inclination of a right-line upon a right-line? For as a right-line and a crooked-line agree as lenghts and in linearineſs and are therein mathematically homogeneal, but as right and crooked differ in kind and have therein heterogeneal figuration; the crookedneſs of the crooked-line having no analogy to the rectitude of the right-line, nor the rectitude of the right line any proportionableneſs unto the curvature of the crooked-line; ſo the inclination of a crooked line upon a right line, and of a right line upon a right line they agree in the common nature of inclination, and the one may be greater and the other leſs as the reſpective ſides fall within, or without: but this relation of greater, or leſs is without any proportionableneſs, and only by the whole kind, depending upon the paſſing of the reſpective ſides, the one within, or without the other. For theſe two inclinations differ ſo far in kind that neither the curvature of the mixt lined inclination hath any thing in it conform, or proportionable to the rectitude of the right-lined inclination, nor the rectitude of the right-lined inclination to the curvature of the mixt lined inclination. In a word, ſo different is the inclinableneſs of a crooked line upon a right line, from the inclinableneſs of a right line upon a right line, that it is impoſſible for the one ever to be either equal, or any way determinately proportionable unto the other: becauſe the coaptation of a right line as a right line, to a crooked-line, as a crooked line is againſt the properties of their figurations, kinds and natures. And for what reaſon ſhould there be leſſer difference between a crooked-lined inclination and a right lined inclination, then there is between a crooked-line and a right line? Yet all this their diſtinctneſs concerns only an heterogeneity in their figuration and not at all, or not primarily their quantities.

The argument, if they be angles, or plane angles, they are homogeneal and of the ſame kind, is of no more force then this conſequence; if they be quantities, or continuous magnitudes, they are homogeneal and of the ſame kind. And they that deny all heterogeneity in angles, becauſe they are all angles; will find it an hard task upon the ſame ground to maintain an analogous homogeneity, or any other conſiderable homogeneity, between right lined angles and ſphærical angles, or any other angles, made by planes cutting the heterepipedal ſurfaces of ſolids, and eſpecially ſolid angles, of what ſort ſoever. And to yield that all plane angles are homogeneal; for it is true: and the moſt abſolute, proper and genuine homogeneity is among plane angles: *i. e.* no part of a plane angle can be any other then a plane angle, how great, or little ſoever, and whether proportionable, or improportionable one to another, *i. e.* whether mathematically homogeneal, or heterogeneal: yet if we ſeriouſly conſider what is this homogeneity which is among plane angles, that all their parts are plane angles, it is not as is ſaid any quantitative, or mathematical homogeneity, the contrary of which is plentifully demonſtrated in Geometry to be poſſible; nor any ſuch homogeneity in reſpect of the manner of their poſiture and figuration, as to exclude all farther diſtinguiſhableneſs in reſpect of fignration; but only denotes that in every plane angle, and in every part of every plane angle, the ſides lye ſtill in the ſame plane: which homogeneity, as is plain, excludes neither heterogeneity, in reſpect of figuration, nor in reſpect of proportion and identity in the way of meaſuring their quantities.

To the objection that in fig. 12. the recto-convexe angle of Contact BAF can be added to the right right-lined angle BAG, ſo making the outer angle of a ſemi Circle FAG; or taken out of it, &c. the recto-convexe angle of Contact BAD out of the ſame right right-lined angle BAG, ſo making the inner angle of a ſemi Circle GAD: and that therefore the recto-convexe angles of Contact FAB and BAD and the right right-lined angle BAG, and the two angles of the ſemi Circle GAD, and FAG, are all of them homogeneal and of the ſame kind; I anſwer. Firſt what need is there of ſuch endeavours, for you to prove their homogeneity, it being Geometrically demonſtrated and confeſſed that there is no analogy, or proportion between them; I mean between the recto-convexe angles of Contact and either the right right lined angle, or either of the angles of the ſemi Circles? And according to your opinion that which is added or taken out is ſaid to be nothing. But eſpecially its thought ſtrange, why there ſhould be ſuch doubting, that heterogeneals can be added and laid up togetheras into one repoſitory; it being with as eaſy connexion performable, as is uſual in the addition of incommenſurables and ſpecious quantities of which it is not known, whether they be homogeneal, or heterogeneal. And out of an heterogeneal ſum, as a ſtore-houſe, why cannot ſome of the heterogeneals be ſubducted, the reſt remaining? and what is more uſual then the adding of heterogeneal figures one unto another? and ſubducting out of a given figure ſome other figure, which is quite heterogeneal to the firſt given figure? So to adde together numbers, and meaſures, and weights? the ſum of which may be divided, multiplyed, increaſed, or leſſened, notwithſtanding its heterogeneity. As ſuppoſing A B, C, D, all heterogeneal, as is uſual in analyticks, the half, or third part, or any proportionable part of this heterogeneal ſum may be given: and any one of the heterogeneal magnitudes ſubducted, the reſt remaining: or a fifth heterogeneal magnitude added to the former ſum: or any Algorythme, ever ſpeciouſly, ſometimes compleatly and abſolutely thereupon performed.

Beſides upon geometrical demonſtration and your own confeſſion, all recto-convexe, convexo-convexe and citradiametral concavo-convexe angles of Contact muſt neceſſarily by your own principles be allowed to be abſolutely heterogeneal to all right-lined angles whatſoever; your ſelf acknowledging that neither in equality, nor in any kind of multiplicity, or ſubmultiplicity is any proportionableneſs poſſible amongſt them. And where between angles a mathematical homogeneity is confeſſed and allowed, yet heterogeneity in reſpect of their Schematiſmes and figurations is undeniable.

The things therefore conſtituting and diſtinguiſhing plane angles in reſpect of their figuration are, as above, their ſides, their inclinations, or rather inclineableneſſes, and their concurrence. That when two angles have all theſe in the ſame reſpective kinds, the angles are upon good reaſon in this ſenſe concluded to be homogeneal: but when between two angles is an heterogeneity in any of theſe things, which are of the eſſence and conſtitution of an angle, thoſe angles may juſtly be judged in this ſenſe to be heterogeneal. And that ſuch a ſpecifick heterogeneity may be in each of theſe, may eaſily be declared as above. As firſt in lines which are the containing ſides, how eaſy is it to diſcover ſuch an heterogeneity? For though a right line, and a crooked-line agree undeniably in the general nature of a line, and of length, and of extenſion, yet the rectitude of the one and the curvature of the other, are ſeveral kinds of poſiture, into which the length of the one, and of the other is diſpoſed: that except, in contrarieties, we can ſee nothing but homogeneity, ſuch an heterogeneity muſt needs be acknowledged between them. And whereas homogeneity, as to ſides, inclination and concurrence, is required to the homogeneal figuration of angles; the heterogeneity of the ſides hinders the poſſibility of ever making them out to be ſuch; or that by any altering their divarication, keeping their preſent properties they can be coaptable. And that angles contained by heterogeneal ſides may be equal, proves only the equality of the inclinations in either, but not the homogeneity of the figuration of the angles, or inclinations; as the equality between a ſquare and triangle in reſpect of their equal perimeters, area's, heights, baſes, proves not in the leaſt the homogeneity of their figures. And as right-lines and crooked-lines are heterogeneal, as above, not poſſibly to be coapted, with the precedent limitations; So alſo are all curve lines, whoſe curvature is unequal and unlike, nay though it may be they be but ſeveral parts of the ſame line, or though the curvature of both, be every way, and every where equal and like, yet if the convexe and concave parts of the one be not alike poſited as in the other, there will be a manifeſt heterogeneity in them, and an impoſſibility of coapting them, obſerving the limitations as above.

And why doth the heterogeneity in the ſides make heterogeneity in the angles, but becauſe thereby is founded an heterogeneity in the inclinations, or rather in the inclinableneſſes of the one ſide to the other? For here, it is not the ſeveral degrees of the ſame kind of inclinations tion that is intended; for then all unequal right-lined angles ſhould be altogether heterogeneal one to another: but it is a more then gradual, a ſpecifick diſtinctneſs in their inclinableneſſes, which we are now diſcovering to make the figuration of the angles more fairly and fully heterogeneal. And as inclination is the habitude of line to line, not being poſited in the ſame right line, nor parallel, for even perpendiculars are in this ſenſe here ſaid to be inclined; ſo, as above, from the heterogeneity of the lines will ariſe an heterogeneity of inclinations: and indeed for no other reaſons do heterogeneal lines make heterogeneal angles, but becauſe their inclinations are neceſſarily heterogeneal. And, as above, heterogeneal inclinations being reſpectively equal, as in ſome right lined and crooked lined angles, this doth not in the leaſt annul the heterogeneity of their inclinations; as a right line and a crooked line may be equal, yet as to the poſiture of their extenſion they are heterogeneal.

And as heterogeneity of ſides, or inclinations makes heterogeneity of angles, ſo likewiſe doth any heterogenealnes in the other point requiſite to the nature of an angle; which is the manner of the ſides concurrence. And there are only three wayes in the concurrence of the ſides of angles, according to which they can be heterogeneal one to another. For either the production of the one concurring ſide becomes coincident with the other concurring ſide, or elſe it departs from it on the ſame ſide on which it did occurre; or elſe it departs from it on the contrary ſide to that on which it did occurre: all which are clearly not ſeveral degrees of the ſame manner of concurrence, but ſeveral kinds of concurrence: *Viz.* the one by way of Contact, the other by way of Section, and a third by way of curvature, or coincidence. That as theſe are diverſified in angles, I mean from kind to kind, not from degree to degree, ſo there is thereby lodged in their figurations an heterogeneity, though in ſome mathematical reſpects, neither ſides, nor inclinations can ſometimes be denyed to be however homogeneal. So particularly angles of Contact in reſpect of their figuration muſt neceſſarily be acknowledged clear of another kind, then all other angles: becauſe the inclination of their ſides is tangent, concurring only in a punctual touch, whereas the inclination of the ſides of all other angles is ſecant, and at the point of their concurrence by reaſon of their inclination they cut one another, or elſe they are coincident; then which, what can make a more material difference in the inclination of the ſides? And as more eſpecially relating to that ſo much urged analogy between right-lined angles and angles of Contact; the inclination of the conteining ſides in every angle of Contact is ſuch as is impoſſible to be between the ſides of any right-lined angle: for the ſides of no right-lined angle can touch without cutting. And what more manifeſt and material difference can be in the inclinations made upon, or unto a right-line, then if in the one caſe a right-line be inclined unto it, and in the other a circumference, or other crooked-line? Yet further to clear that differency of kind, which is between angles of Contact and other angles; I think on all ſides it will be judged unreaſonable to make thoſe angles of the ſame kind; which have neither one common way of meaſuring, nor are coaptable, nor any way proportionable one to another, nor can any way by the contraction, or dilatation of their ſides be made equal one to another; and this we ſhall find to be the condition of many angles one in relation to another. However miſ-underſtand me not, as if I made any commenſurability a full mark of a full homogeneity: for as before crooked-lined and right-lined angles may be equal and of different kinds; having their inclinations different in the kinds of their figurations, though equal in the receſſes of the ſides.

And thus having at large deduced the grand difference which is between the mathematical heterogeneity of angles, and their heterogeneity in reſpect of their figurations, it will now be eaſy for us to extricate our ſelves out of all the difficulties with which former Diſquiſitions upon this ſubject have been involved.

As firſt what is to be underſtood by the equality, which is aſſerted to be between right-lined, and iſoclitical concavo-convexe angles. For it is out of controverſy and on all hands yielded, that to any right-lined angle given may be given alſo a concavo-convexe iſoclitical angle equal; and that alſo, in a thouſand varieties; as is moſt manifeſt, in the circumferences of any two and two equal Circles, or any two and two equal Arches. And ſo in a converſe manner; to any iſoclitical concavo-convexe angle given, whoſe ſides make their receſſe one from the other by an Arch leſs then a ſemi Circle, may be given an equal right-lined angle: although in the infinite number of right lined angles, it is impoſſible to find any more then one right-lined angle equal to the given concavo-convexe angle; becauſe in rectitude there can be no diverſity, as there may and is in curvatures. Now in the above recited caſes, why is equality between ſuch different angles aſſerted poſſible? and what is meant by their equality? and whence, and how is the equality of them to be demonſtrated? Of neceſsity it muſt be founded upon ſome ſpecial method of meaſuring angles, or of ſomewhat which is in ſome, if not in all angles; of which in common both theſe different ſorts of angles are naturally and indifferently capable. And to be ſhort, particular and plain; all the myſterioufneſs of this their equality is founded upon this: that theſe two ſorts of angles, right lined angles, and concavo-convexe angles of equal arches, they both have in common one ſpecial property, of which all other ſorts of plane angles whatſoever are deſtitute. *Viz.* that each in their kind are iſoclitical angles, and the ſides in each are iſoclitical, and in each angle the one ſide by the adduction, contraction, and drawing together of the ſides will be coincident and coapted unto the other. And as the coincidence of right lines the one upon the other makes a right lined angle of Contact impoſſible, ſo the coincidence of iſoclitical crooked-lines the one upon the other makes an iſoclitical angle of Contact impoſsible, except only in an ultradiametral poſiture. And as the menſuration of right lined angles is by the Arches of Circles drawn upon the angular point intercepted between the two iſoclitical ſides, to ſhew how far they are departed from their coincidence; ſo in iſoclitical crooked lined angles, by the ſame way of menſuration an account may be taken of the departure, which each iſoclitical ſide hath made from the other ſince their coincidence: and this is the point in which their equality conſiſts and is accounted, and which founds the mathematical homogeneity which is between them. To inſtance in the caſe which is moſt manifeſt; in fig. 18. from the angular point *A*, let the two arches ABC. and AFH of equall circles conſtitute and contain the iſoclitical concavo-convexe angle CAF, and let the arches ABC and AFH be equall: then thorow the points C and H draw the two right-lines AD and AG. According to what is above delivered; it is on all hands agreed, that the iſoclitical concavo-convexe angle CAH is equal to the iſoclitical right-lined angle CAH: as is copiouſly demonſtrable from the equall arches of Circles drawn upon the angular point as center, cutting all the four lines: *viz.* the Arches comprized between the two iſoclitical crooked-lines, are ſtill equall to the reſpective arches comprized between the two iſoclitical right-lines. For example in the chord AH take any where at pleaſure the point I and from the center A draw the arch IB cutting the arch AFH in the point F and the right-line AEC in the point E and the arch ABC in the point B. The arch BF between the two iſoclitical arches ABC and AFH is ſtill equall to the arch EI intercepted between the two right-lines DA and GA. For the arches ABC and AFH being equall in equall circles, the right-lines AC and AH are equall: and al-

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ſo AE ſemidiameter is equall to AI ſemidiameter: and by the converſe of the ſame ratiocination AB arch is equall to AF arch: ſo as in ſhort by ſuperpoſition, or adaptation the arch BE will appear to be equal to the arch FI; and therefore adding the common arch EF; the arch BF intercepted between the two iſoclitical crooked-lines ABC and AFH is equall to the reſpective arch EI intercepted between the two iſoclitical right-lines: and this whereſoever the point I be taken in the right-line AH. So as by this common way of menſuration, common to both theſe ſorts of angles, by reaſon of the iſocliticalneſs, and the coaptability and coincidibleneſs of the ſides in each, the one being an iſoclitical concavo convexe angle is copiouſly demonſtrated to be equal to the other being a right-lined angle. But now after what manner are we to underſtand this equality aſſerted between ſuch right-lined and iſoclitical concavo-convexe angles? It is not an every way abſolute equality which is between the angles, ſuch as is between two equall ſquares, or two like and equall triangles, or any two regular and equall figures of the ſame kind or, to come nearer to the matter, it is not ſuch as is between two equall right-lined angles, or between two equall, iſoclitical, concavo-convexe angles, all whoſe four ſides are all of them iſoclitical each in reſpect of all the reſt:

but as things that are like each other, are like only in ſome things, and unlike it may be in many others; ſuch is the equality between any two ſuch angles; *Viz.* only a reſpective equality, ſuch as is poſſible among heterogeneals, and inferring a neceſsity of ſome other reſpective inequalities. And ſuch an equality may be between two mere heterogeneals; they may be of equal length, and different breadth, or weight: ſo a Triangle, and a ſquare and a Circle may be all equal, either in perimetry, or ſurface, but not in both; ſo a right-lined angle, and an iſoclitical concavo-convexe angle may be equal in reſpect of the receſſes which the iſoclitical ſides make each from other and from their coincidence and coaptation, but in other reſpects they want not their manifeſt inequalities and heterogenealneſs. *As* a ſolid to a ſolid may have equal proportion that a line to a line, yet ſolids and lines are heterogeneal: ſo a crooked-line from a crooked-line may make equal receſſes, as a right-line from a right-line, and yet in many other things much heterogenealneſs may be in the angles which they conſtitute. You will ſay wherein? I anſwer in the rectitude and curvature of the containing ſides. *And* in theſe different reſpects two iſoclitical concavo-convexe angles may be both equal and unequal the one unto the other, *Viz.* equal in the receſſes of the ſides, but unequal in the curvatures of the ſides: in the ſame manner as two figures may be equal in their perimetry, or ſuperficial, or ſolid content, and yet be figures of different kinds under diverſe inequalities: as the one a Rhombus, the other a ſquare, the one a Cylinder, the other a Dodecaedron. So a thouſand concavo-convexe iſoclitical angles may be equal in reſpect of the receſſes of the ſides, yet each of a ſeveral kind: as a thouſand figures different in kind may be equal in perimetry, height, baſe, ſuperficial, or ſolid content. But you'll ſay, what is the rectitude, or curvature of the containing ſides to the nature of angularity? I anſwer, they are of eſſential concern to the limiting and determining the nature of angles: angularity being the habitude of concurring lines each in reſpect of the other, as to their concurrence and inclination. And though the inclination of iſoclitical crooked lines may be equal to the inclination of right-lines one upon another, in reſpect of the equal receſſes and departures which the iſoclitical lines make each from the other; yet there ſtill remains a vaſt inequality, diſsimilitude, and unanalogableneſs between the angles and their inclinations, in reſpect of that little of figuration, without which neither can an angle be conſtituted, nor an inclination made: in a word the ſides may make equal receſſes, yet be unequal in their curvature, and unlike in their figuration: and neither by imagination, nor circumduction, nor any other operation can the one poſsibly be reduced, or coapted to the other, without ſetting the homologal points at improper and undue diſtances and poſitures one from another; which ſhews a ſpecifical difference between the two inclinableneſſes of the one and the other: beſides that a right-lined angle can continue its inclination between the ſides infinitely; but many iſoclitical concavo-convexe angles thereunto equal by the neceſsity of their curvature muſt terminate within a very little ſpace: circumferences and ſeveral other arches, not being poſsible to be produced beyond their integrity; ſo as ſome three given angles conſtituting a given triangle as to its angles, cannot, in like manner, conſtitute a triangle of any given magnitude; which is otherwiſe in right-lined angles. And that the equality between iſoclitical concavo-convexe and right-lined angles is not ſo abſolute as to make them every way alike, equal, and of the ſame kind, may appear eſpecially in this, which is elſewhere demonſtrated; *Viz.* that an ultradiametral concavo-convexe angle of Contact, being iſoclitical, is alwayes equal to two right right-lined angles, which no one right lined-angle can be: and if it be aniſoclitical of the larger, it is ever greater then two right

right-lined angles can be, which is impoſſible alſo for one right-lined angle to be.

And the difference between mathematical heterogeneity and the heterogeneity of angular figurations being as above diſcovered; the nature as well of aniſoclitical crooked-lined and mixt-lined ſecant angles will as clearly appear: *Viz.* that comparing them with right-lined angles they are compound and concrete angles conſtituted of right-lined angles and angles of Contact; which are demonſtrated every way heterogeneal: and ſuch aniſoclitical ſecant angles cannot be divided into any number at pleaſure of parts homogeneal either mathematically, or in reſpect of their figuration, but of neceſſity ſome of them muſt be both wayes heterogeneal. This is manifeſt, becauſe the fluxe, or circumduction of angles of Contact or of one of their conteining ſides addes only a right-lined angle to them: after the ſame manner as the four right right-lined angles, which compleat the ſpace in any plane, about any given point, may be exhauſted by the circumduction of a crooked, concave, or convexe autoclitical, or antanaclitical line, as well as by the circumduction of a right-line. And that this ties not both angles to be of the ſame kind, may eaſily appear from the heterogeneity between lines and their fluxes, which are ſuperficial; or ſurfaces and their fluxes, which are ſolid. No wonder therefore if by the fluxe of an angle of contact, or of one of its ſides, be created another kind of angle, holding no analogy with the former: the heterogeneity and improportionableneſs of right-lined angles and angles of Contact having been demonſtrated.

So upon the ſame ground we may be aſſiſted to look into the ſpecial properties, conſiderable in the ſeveral kinds of angles peculiar unto ſome and incommunicable unto others, for example.

In right lined angles, neither the greateſt poſsible angle, nor the leaſt poſsible angle can be given; though all uſually ſaid to be within the compaſſe but of one kind. But to purſue the difference which is in angles; angles of Contact, except ſuch as are contained under lines of the ſame rectitude and curvature are every one of a ſeveral kind, either mathematically, or in reſpect of their figuration, or both. And except convexo-convexe angles of Contact of equal arches, which may be divided into two equal angles by common right-lined tangents; all other angles of Contact are, every one, both the greateſt, and leaſt poſſible, of their ſpecial kind: and every angle of Contact contained under the convexe ſide of its arch, or arches, is the leaſt poſsible under thoſe ſides; which I ſuppoſe was the ſpeculaculation unhappily miſſed by thoſe learned men, who would have impoſed upon the world, upon that their miſtake, the dream of the coincidence of the ſides in ſuch angles.

Again a right-line may be drawn dividing a convexe-convexe angle of Contact, whether it divide it equally, or unequally, but a right-line cannot be drawn dividing either of the recto-convexe angles of Contact into which the former was ſo divided; whether the two recto-convexe angles of Contact be of the ſame, or different kinds: as is up and down demonſtrated in Geometry.

In citradiametral concavo-convexe angles of Contact, either the *A*rches are of unequal curvature, or which is tantamount, though they be of equal curvature, yet they touch not at homologal and anſwering points, being not all over of equal curvature; which makes them notwithſtanding the reſpective but not anſwering equality of their curvature to be aniſoclitical. And between their Arches containing the angle of Contact, a right-line cannot be drawn; but infinite crooked-lines in number may be drawn, bearing in like manner their convexity towards the concave which is inward, and their concavity towards the convexe of the other ſide, which is alſo inward.

Concavo-convexe angles concurring by way of ſection, and having two right-line tangents drawn upon the Arches at the angular point are equal unto the right-lined angles contained under thoſe right-lined tangents; adding reſpectively to each right-lined angle one of the recto-convexe angles of Contact, and ſubducting out of it the other recto-convexe angle of Contact: and when thoſe two recto-convexe angles of Contact are equal, as they are, when the ſides are iſoclitical, then the concavo-convexe angle is exactly equal to the right-lined angle: but when the two recto-convexe angles of Contact are unequal, as they are when the ſides of the concavo-convexe angle are aniſoclitical then the concavo-convexe angle and the right-lined angle are unequal.

Concavo-concave angles concurring by way of ſection (as all ſuch ever do; or by coincidence, and then one right-lined tangent gives the analyſme of them, ſhewing the two recto-convexe angles of Contact, by which the angle of coincidence is leſs then two right right-lined angles) are by two right-lined tangents at the angular point reduced into the right-lined angle; which is the leaſt of thoſe right-lined angles that are greater then it, exceeding it only by two recto-convexe angles of Contact to be taken out of it.

Convexo-convexe angles concurring by way of ſection are by two right-lined tangents at the angular point reduced into a right-lined angle; unto which to make it equal to the convexo-convexe angle, are to be added two recto-convexe angles of Contact. And the right-lined angle is the greateſt of all the right-lined angles that are leſs then the convexo-convexe angle.

Recto-concave angles concurring by way of Contact are the greateſt angles poſſible under thoſe two ſides.

Recto-convexe angles concurring by way of Contact are the leaſt angles poſſible under thoſe two ſides.

If we compare recto-concave angles of Contact with right lined angles, they are leſs then two right right-lined angles by one only recto-convexe angle of Contact. And the leaſt recto-concave angle of Contact is greater then the greateſt right-lined angle whatſoever.

Recto-concave angles concurring by way of ſection compared with right-lined angles which are conſtituted, *i. e.* compleated by the right-lined tangents drawn upon the Arches at the angular points; are leſs than ſuch reſpective right-lined angles by a recto-convexe angle of Contact.

Recto-convexe angles concurring by way of ſection compared with right-lined angles which are conſtituted, *i. e.* compleated by right-line tangents drawn upon the Arches at the angular points; are greater then ſuch right-lined angles by a recto-convexe angle of Contact.

Every angle of curvature, or coincidence, having a right-line tangent drawn upon the angular point, appears to be leſs then two right right-lined angles by two recto-convexe angles of Contact.

The inclination of the ſides without the angular point at any two reſpective, or other points, the one taken in the one ſide, the other in the other, is very nearly ſhewn, and as nearly as is poſsible in right-lines, by the right-line tangents of thoſe reſpective points: but in mixed lined and mixed crooked-lined angles by ſeveral wayes of accounting, ſeveral points are made to anſwer one another, as by accounting by diſtance from the angular point, or by accounting by equalneſs of lines along the ſides &c.

Mixed lined angles of Contact, when they can be, and are, divided by a right-line, the parts are heterogeneal and unequal: and one of the unequal parts is a right-lined angle.

Every recto-convexe, and convexo-convexe, or citradiametral concavo-convexe angle of Contact is the leaſt poſſible under thoſe ſides.

Rectilineary menſurableneſs in mixt lined, crooked-lined angles concurring by way of ſection, begins from the recto-convexe angle of Contact: as in right-lined angles from coincidence.

A convexo-convexe angle of Contact, in reſpect of dividableneſs by right-lines is an angle made up only of heterogeneal parts, when it is a mixt-crooked-lined angle: but when it is an unmixt crooked-lined angle, it hath ſome parts which are homogeneal, *Viz.* two equal recto-convexe angles of contact, which are therein added the one unto the other. And thoſe two equal recto-convexe angles of contact, as they are homogeneal, I mean of the ſame kind one with another, both mathematically and in reſpect of their figuration; ſo mathematically they are homogeneal and of the ſame kind with the convexo-convexe angle which was divided; but in reſpect of it, as to their figuration, they are heterogeneal and of another kind.

The moſt ſimple angle may be divided into heterogeneal parts: *i. e.* the inclinableneſs of the one ſide to the dividing line both in reſpect of figuration and proportion may be ſpecifically different from the inclinableneſs of the other ſide to the ſame dividing line: as a pentagone may be divided into a tetragone and a triangle, ſo a recto-convexe angle of Contact may be divided into two parts heterogeneal the one to the other, and to the firſt angle of Contact, both in reſpect of figuration and proportion:

viz. into a new recto-convexe angle of Contact, and a concavo-convexe angle of Contact. Therefore no angle can be ſaid homogeneal in that ſenſe, as if it could not be divided into parts heterogeneal; whether you pleaſe to underſtand it, in reſpect of mathematical homogeneity, or poſiture, and figuration, or what reſpect ſoever elſe that limits and diſtinguiſheth plane angles one from another

And to give a brief and general account of the comparative admenſuration of angles, as not being right-lined, yet by way of comparative admeaſurement, they may in reſpect of their rectilineary parts be reduced and referred to thoſe that are right-lined; the containing ſides not being right-lines, at the angular point draw right-line tangents touching the arch, or arches in the angular point; and the right-lined angle contained by thoſe right-lined tangents will be, as to the receſſes of the ſides at the angular point, either equall unto the firſt propoſed angle, or the leaſt right-lined angle greater then it, or the greateſt right-lined angle leſſer then it: or if two right-lined tangents cannot be thus placed at the angular point, either the firſt propoſed angle was a mixt-lined angle of contact, ſaid, if a recto convexe to be leſs, if a recto-concave to be greater then any right-lined angle; or elſe it is a crooked-lined angle of contact, which if convexo-convexe, or concacavo-convexe and citradiametral is leſs then any right-lined angle, but if concavo-convexe and ultradiametral, is greater then any right-lined angle, nay ſometimes equall to, or greater then two right right-lined angles: or elſe it is an angle of coincidence, or curvature. All which is to be underſtood to ſhew the inclination of the ſides at the angular point, as the chief for uſe in Geometry, but not neceſſarily elſe-where. So crooked-lined, or mixed lined angles are compared with right-lined angles by drawing at the angular point right-lines touching the Arches there, and comparing the crooked or mixed lined angle with the right-lined angle ſo conſtituted, reſpectively adding, or ſubducting the recto-convexe angles of Contact hereby created: and this, whether the Arches be iſoclitical, or aniſoclitical, or however poſited. So all crooked, or mixed-lined angles concurring by way of ſection may have a right-lined angle given, which if it fall ſhort of equality is either the leaſt of the right-lined angles that are greater, or the greateſt of the right-lined angles that are leſs then the firſt crooked-lined, or mixed lined angle. And ſo an analyſme may be made of the greater angle into its heterogeneal parts; and the crooked lined, or mixed-lined angle may be reduced unto, or compared with right-lined angles, only with the addition, or ſubſtraction of recto-convexe angles of Contact, being angles leſs than the leaſt right-lined angle whatſoever.

All angles have their inclinations compounded of the inclinations of the interjacent lines each to other in order: and of the inclinations of the ſides to the lines next adjacent to them; which compoſite inclination may be heterogeneal, as well as homogeneal in reſpect of the inclinations of which it is, or may be compounded.

Equally arched convexo-convexe, or concavo-concave angles, may by a right-line be divided into equal parts mathematically homogeneal, but heterogeneal in reſpect of their figuration: but ſuch angles cannot be divided into any more, or any other equal parts, for the reaſon immediately to be ſubjoyned.

Heterogeneals taken together in ſeveral concretes proportionably, *&c.* each reſpectively in the ſame proportion, they hold exact proportion, concrete to concrete: as double cube and double line, are double, to ſingle cube and ſingle line, *Viz.* the concrete to the concrete: but ſet them out of the ſame reſpective proportions and the concretes are no way proportionable, or in analogy, concretely to be compared: as double cube and treble line, are in no proportion, to ſingle cube, and ſingle line. So double number, and double weight, and double meaſure, the whole concrete, is double, to ſingle number, ſingle weight and ſingle meaſure: but ſetting them out of the ſame reſpective proportions; double number, and double weight and treble meaſure, being alltogether concretely taken, are mathematically heterogeneal aud improportionable to ſingle number, ſingle weight and ſingle meaſure; being in like manner concretely taken: becauſe the heterogeneals in the one concrete hold not the ſame reſpective proportions to the anſwering heterogeneals in the other. So convexo-convexe, or concavo-concave equally arched angles being ſecant, hold proportion when divided equally, as they may, by right-lines: but they are merely heterogeneal and without proportion, when divided by a right-line unequally. The ground of which is the heterogeneity of the parts, of which ſuch concrete angles are made up when compared with angularity conſtituted by right-lines; which heterogeneal parts, when the angle is divided equally in two by a right-line, are in the concretes, each reſpectively in the ſame proportion; ſo making the concretes, though of heterogeneal parts, to be mathematically homogeneal and proportionable one unto the other: but when the angle is unequally divided; in the two concretes the heterogeneal parts, of which they are made up, are not reſpectively in the ſame proportion; for the recto-convexe an-

angles of Contact, in the concrete angle, are divided equally, and the right-lined angle, which is in the concrete angle is divided unequally: ſo making the parts of the divided angle mathematically heterogeneal and improportionable; becauſe the compounding heterogeneal parts are not reſpectively according to the ſame proportion divided.

The leaſt poſſible angle under any two given incoincidible lines is the leaſt angle of Contact which is poſſible under them.

Recto-convexe, and citradiametral concavo-convexe angles of Contact as they are the leaſt angles poſſible under their ſides; ſo they are indiviſible into parts holding all each to other any thing of mathematical homogeneity, or proportionableneſs: Like unites they may be multiplyed to any proportion as whole numbers, and ſeparately ſet: but cannot at pleaſure be divided, nor at all into parts which are all of them mathematically homogeneal: nor can any number of them be at pleaſure adjoyned one to another. So to any right-lined angle another may be imagined in any proportion, but ſometimes it muſt be the compoſition of ſeſeveral angles, and more then can ſtand at the ſpace circumjacent about any one point. It not being poſsible for above four right right-lined angles to ſtand about the ſame point. As to a given point, other points may be given in any proportion , as whole numbers ; but they muſt not then be adjoyned one to another : but properly by the dividing of a point, farther proportions are not to be expected. So in ſuch angles , though heterogeneal diviſions be infinitely poſsible ; homogeneal , I mean mathematically , being impoſsible , are not, as above, to be expected.

Two unequal aniſoclitical angles contained under the ſame two aniſoclitical ſides , though in reſpect of their figuration , both are formed upon the ſame inclinableneſs of the ſides one to another , yet they are mathematically heterogeneal : becauſe being two concrete angles made up of heterogeneals , *viz.* of the angle, or angles of Contact and right-lined angles ; in the two concrete angles , the angles of Contact are in the proportion of equality , and the right-lined angles in the proportion of inequality : ſo as the concrete angles can have no proportion the one to the other.

The half of the inclinableneſs of any arch upon its ſelf, *i. e.* upon another arch like and equal, is ſtill comprehended and contained under the inclinableneſs of a right-line upon the ſame arch : that though their figurations are ever heterogeneal , and their inclinations can never be equal , yet they may be mathematically homogeneal : as different numbers are alwayes unequal though ever proportionable.

To no aniſoclitical angle, whether of Contact, or concurring by way of ſection can any right-lined angle be made equal. And generally between iſoclitical and aniſoclitical angles equality is impoſſible. And when aniſoclitical angles are compared with thoſe that are iſoclitical as greater, or leſſer; it is not to denote in both any mathematical homogeneity, and that ſo by poſſibility they are reducible to a true, compleat and analogous equality; but the intent is only to declare, whether of them hath, or can have the containing ſide, or ſides falling within, or without the other; which is only from their inequality by the whole kind. So though aniſoclitical angles may be greater, or leſs then a right-lined angle, yet between the one and the other there is neither common way of meaſuring, nor any proportion, though both of them be quantitative plane angles. There is no common way of meaſuring them, becauſe of the aniſocliticalneſs, and if there could be any proportion between them, then might a right-lined angle be given equal to ſuch aniſoclitical angles; the contrary of which is otherwiſe clearly demonſtrable. So as a thouſand angles quantitative by confeſſion and having right-lined angles leſſer and greater then themſelves, yet can have no right-lined angle equal to them.

A right-lined angle cannot have its inclination at the angular point, much leſs all along the ſides divided equally by a crooked-line, whether autoclitical, or antanaclitical, or of what curvature ſoever: though any ſuch crooked-line may from any angle divide equally the plane bounded within a right-lined triangle: for if in fig. 17. BAC be a right-lined angle: I ſay, no crooked-line &c. can divide it at the angular point equally. If it be poſſible let it be divided into equal parts by the arch AGE whether autoclitical, as when it is conceived to be part of the arch AGEF, or antanaclitical, as when it is conceived to be part of the arch AGEHD. Then let the right-line AD divide equally the right-lined angle BAC. It is manifeſt, if the crooked-line AG never occurre with the right-line AD, that then it doth not divide the angle BAC equally; for the right and crooked-lines cannot be coapted. Let therefore, if they do occurre, the place of their firſt meeting, or occurrence be at E. Therefore the arch AGE falleth within the angle BAD and therefore divides the whole angle BAC unequally, whether it be autoclitical, or antanaclitical. And by the ſame demonſtration appears that it is as impoſsible for it to divide the inclination all along the ſides equally. And though a crooked-line may divide any right-lined figure, becauſe of its perfect bounds, from any angle into equal parts ; yet this no way evinceth any poſsibility of dividing a right-lined angle by a crooked line into equal parts : and though from any point of ſuch a dividing crooked line, lines might be drawn making up a figure , whoſe parts divided by the crooked line from the angle are equal ; yet this is no more then is performable upon any point of any crooked line drawn between the ſides of a right-lined angle at randome , whether the parts of the angle be homogeneal , or heterogeneal , equal , or unequal.

Difference of curvature by a perpetual neceſſity infers difference of inclination , whether the curve line be inclined upon a right-line , or upon a curve line ; for ſtill the one of the curve lines will fall within , or without the other. So let inclination upon a right-line be in a recto-concave angle of Contact , recto-convexe angle of Contact , recto-concave , or recto-convexe ſecant angles ; by no right-line can any of thoſe inclinations be made upon the firſt right-line , nor by any other crooked-line : but ſtill the lines will fall either within, or without. And all angles of Contact under lines of different curvature and rectitude , as they are of a thouſand Mathematically and extra-quantitatively different kinds one from another , ſo they are as manifeſtly diſtinct in kind from all right-lined angles whatſoever.

And as every number is a different kind, in like manner as to poſiture and figuration is every line a diſtinct kind differing from all other not agreeing with it in rectitude and curvature; whether the curvatures be homogeneal, *i. e.* every where equal, as in Circles, or heterogeneal, *i. e.* unequal in the ſeveral parts, as in ellipſes, hyperbolas, parabolas, &c. And accordingly judgment is to be made of the ſides under which angles are contain'd, and the angles contain'd under them: alter the kind of either ſide, and the kind of the angle is chang'd; ever, in reſpect of figuration; and moſt what, mathematically; becauſe the former inclinablenes of the ſides is taken away, and a new kind of inclinablenes introduced between them. Hence appears; no two recto-concaves, no two recto-convexes, no two concavo-convexes, no two convexo-convexes, being all angles of Contact can be equal, except their ſides have the very ſame rectitude and curvature. In general, equality is not to be aſſerted between angles, except either for the menſurablenes of both in ſome common way of meaſuring, or at leaſt becauſe they can be ſo caſt into a coaptation, as that either ſhall contain all the quantity which is in the other. And as he erred in ſquaring the ſuperficial content of a circle, that in his quadrature left out a ſmall lunular figure; ſo neither can he be juſtifyed to have given one aniſoclitical angle equal to another, who, as he muſt of neceſſity, takes in, or leaves out an angle, or ſome angles of contact which are in the one and not in the other.

Particularly no mixed lined, nor mixed crooked lined angles whatſoever, whether ſecant, or of contact, can by any poſſibility, either by right-line, or crooked line be divided into equal parts, or angles, whether of the ſame, or of different, kinds. And in few, thoſe angles, which cannot be divided into two equal parts cannot be divided into three, fower, five, or any other number of equal parts.

The comparative admenſurement, as above, of right-lined and not right-lined angles is as well of their heterogeneal inequality, when heterogeneal, as of their homogeneal equality and proportionablenes when homogeneal.

All not right-lined angles comparatively admeaſured, to right-lined angles, only with the addition, or only with the ſubduction of any angle, or angles of contact, are heterogeneal to all right-lined angles, as alſo, if when one angle of contact is to be added, and another ſubducted, the two angles of contact be unequal. And as is manifeſt, of ſeveral not right-lined angles, that is ſtill the greater, whoſe comparative and relative admeaſurement is made to the greater right-lined angle: and of thoſe, whoſe comparative admeaſurement is unto equal right-lined angles, they are greater, or equal, or leſs according to the equality, or inequality, and comparing together of their angles of Contact. And of concavo-concave angles the right-lined angle cannot be given, which is the greateſt of thoſe right-lined angles that are leſs then the concavo-concave angle: nor in convexo-convexe angles can be given a right-lined angle, which is the leaſt of thoſe right-lined angles which are greater then the convexo-convexe angle: nor in recto-concaves the greateſt of the leſs: nor in recto-convexes the leaſt of the greater: whereas in concavo-convexe angles, the right-lined angle to which the comparative admeaſurement is made, may according to the caſe be either equal, as when the ſides are iſoclitical; or the greateſt right-lined angle that is leſs, as when the concave ſide is of leſs curvature; or the leaſt of the greater, as when the concave ſide is of greater curvature.

From theſe things may appear how that objection is to be anſwered, in which you urge that there is a proportion between mixed lined angles, whoſe ſides are ſecant, and right-lined angles, becauſe the one by its multiple may exceed the other; and that therefore an homogeneity, mathematically to be underſtood,

ſtood, is to be acknowledged amongſt them. To this is anſwered that all aniſoclitical angles whatſoever having their ſides concurring by way of ſection when compared unto right-lined angles are heterogeneal compoſitions of angularity, *i.e.* cannot be divided into any number at pleaſure of parts all equal; but as is manifeſt, they in that their relative nature are concrete and compoſite angles, formed of right-lined angles by adding to them, or ſubducting from them mixed lined angles of Contact: ſo as the right-lined part of the ſecant aniſoclitical angle, or any the leaſt part of it hath true proportion with all right-lined angles; and the whole being heterogeneal, yet hath a ſemblance of proportion with right-lined angles, becauſe of thoſe parts which it hath which are homogeneal with them; but between the other heterogeneal part which is the mixt-lined angle of Contact, and right-lined angles, neither is, nor can be any proportion: and it is becauſe of this part that it is not a true, but a ſeeming proportion only, which is between the whole concrete heterogeneal, compoſite aniſoclitical angle, and right-lined angles: for if it were a true proportion and proportionablenes which were between them, it ſhould then be poſſible to give a right-lined angle equal to ſuch an aniſoclitical ſecant angle; which when done, I have no more to ſay, being well aſſured I am able to demonſtrate the contrary upon whatſoever right-lined angle ſhall be offered under that notion. And whereas it is urged that a right-lined angle is, and may be equal to a mixed lined angle; all their difference being only in this, that the ſides of the one are more ſpread and divaricated then the other; what is this elſe but to ſay, that they are equal, ſaving that the one is bigger then the other. For angles of Contact can divide angularity, or ſpace equally, or unequally into more, or fewer parts: and they are not indiviſible, becauſe every one contains innumerably more in it: and by the definition of an angle, lines have a ſufficient inclination to conſtitute an angle, if in the ſame plane, they lye not both in the ſame right-line. And to all other caſes and objections of the like nature grounded upon them athematical heterogeneity of the parts, of which ſuch ſpecial angles are conſtituted in reſpect of right-lined angles, like anſwers may be addreſſed.

Though a right-lined angle cannot be divided into equal parts of the ſame kind mathematically, or in ſhape by a crooked-line, nor a mixed, or crooked-line angle by a right-line into parts of the ſame kind in reſpect of figuration; yet ſome crooked lined angles may be divided into parts exactly equal of the ſame kind one with another, and with the whole ma-

mathematically by a right-line. For it is apparent, there may be between magnitudes a ſufficient homogeneity for proportionablenes without excluding all further diſcriminableneſs between them: as between equal arched convexo-convexes, or concavo-concaves and their parts when they are equally divided. So between lines of all kinds, and numbers of all kinds, is proportionality. Yet ſtill their proportionality is in reſpect of ſomewhat which is homogeneal in them. *Viz.* that they are all reſoluble into parts that are homogeneal, or parts into which the reſt are homogeneally reſoluble, or after the ſame manner menſurable, as equal arched convexo-convexe angles into two equal recto-convexes: yet notwithſtanding the proportionablenes between the convexo-convexes and the recto-convexes; ſuch an heterogeneity is in their figuration, that by no divarication of ſides can they ever be made equal. And though the equal arched convexo-convexes and concavo concaves may be divided by a right-line into two equal parts; by no lines whatſoever can they be divided into any more equal parts nor their recto-convexes by any line whatſoever into two equal parts; ſuch parts of ſuch angles being more impoſſible to be given in Geometry, then the ſquare roots of unſquare numbers, or cubicke roots of uncubical numbers are in Arithmeticke. As any two points, ,or any two indiviſibles may have ſome few proportions amongſt them, but no more: the latitude of proportionablenes being limited by the ſpecial natures and kinds of things: ſo between ſpecial kinds of numbers cannot be all proportions: nor between ſpecial lines all angles, or all proportions in all angles.

Hence alſo we may clear the poſſibility of that ſpeculation of the learned perſons by you named, that in heterogeneals there may be a paſſing from greater to leſs, and from leſs to greater in a continuous manner without, ever paſſing thorow equality; however you are pleaſed to beſtow upon it a contemptuons ſmile, if not an hiſs: *Viz.* that we ſhould aſſert that mixed lined, or crooked lined angles can paſs by the divarication of the ſame ſides from being greater to be leſs then given angles, or contrarily by their contraction, and yet in that tranſit, never be equal. What is more obvious then to give inſtances of aniſoclitical angles, leſs then a given iſoclitical angle, and by divaricating and diſtending the ſides of the aniſoclitical angle, it may be made to exceed the firſt given iſoclitical angle; yet in all the way they could never be equal the one to the other; the one being iſoclitical, the other aniſoclitical: coaptation and the paſſing of the ſides from the angular-point every where prove their inequality: and let thoſe that aſſert their equality any way ſhew and admeaſure it. And the ſober underſtanding of thoſe ſayings, that ſuch angles paſſe from leſs to greater without ever being equal ; is not to aſſert any homogeneity, or proportion of any kind between them, or common way of meaſuring their quantity in and according to the ſame indefinite quantity, but only to ſhew how the ſides may paſs within and without each other; but becauſe of their aniſocliticalnes and difference in reſpect of rectitude and curvatures, they can never be brought to be coincident. And why ſhould this appear ſo monſtrous, that tranſitions ſhould be, in the ſenſe aboveſaid, made from greater to leſs, without paſſing thorow equality? ſeeing it is moſt manifeſt that though a crooked-line circumducted about the angular point of a right-lined angle makes infinite diviſions of the right-lined angle; yet they are ever unequal, and never by poſsibility can be equal: though notwithſtanding, the ſame crooked line may divide a crooked lined iſoclitical angle eqnal to the given right-lined angle into equal parts; which beſides plentifully ſhews the heterogeneity which is between equal right-lined and crooked-lined iſoclitical angles, notwithſtanding their equality: yet by making up the right-lined angle into a compleately bounded figure, the crooked-ked-line may divide the plane of it into equal iſoepipedal and iſorrhopical parts: as is more eaſy to demonſtrate then that there ſhould be any need to ſet it down. And it is not to be ſtranged at, that we aſſert a crooked-line can divide a right-lined triangle into equal parts, but not any of its right-lined angles: for the figuration of the one is compleat, and the production of the crooked-line, as well as of the lines containing the right-lined angle, in the triangle are limited; all which are quite otherwiſe in mere angles, being in many kinds of angles and inclinations very alterable by the production of the lines. And if you will pertinaciouſly ſay that a crooked-line may divide equally a right-lined angle, ſhew their equality, and your way of admeaſuring the equality of the parts: both coaptation of ſides, in whoſe habitude the nature of angles chiefly conſiſts, and alſo the way of meaſuring by interjected arches demonſtrating and declaring the contrary: Nay ſometimes in quantities among which there is true proportion, the ſpecial differences of their kinds may be the authour of little leſs. So commenſurable quantities, being in their commenſurability infinitely diviſible, may be continually increaſed, or leſſened by quantities vaſtly, leſs then any given quantity; yet as they paſs from leſs to greater never can hit equality with any the like quantities of the ſame general kind, being incommenſurable. So in the preſent queſtion, heterogeneity of ſides makes an impoſſibility to coapt, though by ſtraitning and divaricating they may fall within, or without, and ſo be leſs, or greater, but being impoſſible to coapt, and incapable of any other common way of meaſuring, equality cannot be conceived in them; in which is præſuppoſed a common way of meaſuring to declare and prove their aſſerted proportion and equality. And in this whole matter what is more ſaid then that the cutting inclination of a right-line upon a right-line, and of a right-line upon a crooked, or of a crooked-line upon two crooked-lines being all of different curvature, may any of them be greater, or leſs then one another heterogeneally, but can never be equal; as to thoſe that will conſider, is moſt clearly manifeſt and neceſſary. And this is no more ſtrange, then that odde numbers may be leſs, or greater then any even number, at leaſt above two, yet by their conſtitutive nature, they can never be equal: and for the ſame reaſon magnitudes commenſurable and incommenſurable may be greater, and leſs then one another, and by a leſs quantity then any quantity that can be given; yet, for the ſpecialty of their natures, they can never be equal, remaining ſo diſtinct in their ultimate kind: and however they have many general things in which they agree, yet they are not accountable ſpecially, but by ſeveral wayes and algorythmes. And as it is the binding up of numbers to the ſpecifick properties of Evenneſs, or oddneſs, and of magnitudes to commenſurability, or incommenſurability, that makes equality between their kinds impoſſible; ſo it is the incoaptablenes and want and impoſſibility of a common way of meaſuring, which is between the two and two ſides of ſuch heterogeneally unequal angles, keeping all along the properties of their inclination, that renders equality between their kinds impoſſible; though either may ſometimes truly and manifeſtly be greater or leſs then the other, however without proportion. For in heterogeneals may be a two fold inequality, either an inequality according to proportion, or an inequality without proportion: and though they be heterogeneals, if it be a true limited, determinate proportion of inequality which is between them, as between an odde number and an even, a commenſurable magnitude and an incommenſurable, they are then conſidered as ſome way, or according to ſomething which is, homogeneal, not heterogeneal, in both: but if their inequality, though real and apparent be without any true and homogeneally determinable proportionablenes, as between right-lined and aniſoclitical angles, then the conſideration had of them is in their heterogeneity. So to the objection, that right-lined angles, and recto-convexe angles of Contact have proportion of greater and leſs, and are therefore homogeneal, if any quantitativenes be to be aſſerted in the recto-convexe angles of Contact; and ſo of the reſt: I anſwer, there is not a proportionable inequality between right-lined angles and recto-convexe angles of Contact, and the like; but an inequality, which is improportionable: as when the Earth is ſaid to be intruth greter then a point; though in many particular hypotheſes contrarily conceived, for the better obſerving and accounting ſeveral phænomena, and the better accommodating of inſtruments for the making of obſervations. So in the geneſis of quantities and figures, when one quantity hath its geneſis by the fluxe, or motion of another, the quantity formed by that motion is greater then the quantity moved, though ſtill without any proportion: ſo by the divaricating of the ſides, by the motion of one of the ſides, of a recto-convexe angle of Contact, is a geneſis of recto-convexe angles of ſection; and the recto-convexe angles of ſection are ſtill greater then the recto-convexe angle of Contact, or its reſpectively moved ſide: and all ſtill without any proportion. And inequality without proportionality being ſo uſually aſſerted,

ted, and ſamiliar between mere heterogeneals, doth eaſily acquit it ſelf from the reproach you charge it withal, of being a contradiction in the very terms; for finite and infinite never were denyed to be unequal, yet never can be made out to be mathematically homogeneal and proportionable. For though all proportion is either of equality, or inequality; that hinders not but an inequality in heterogeneals may be admitted, without an aſſerting of proportion between them. So as mathematical homogeneity is not proved by inequality, till proportionablenes be as well proved as inequality. Of the ſame leaven is that ſtrange kind of reaſoning you uſe ; to any right-lined angle a crooked-lined angle may be made equal (we confeſs a thouſand ſeveral crooked-lined angles may be made equal to any one right-lined) angle) but you from thence inferre that therefore all crooked lined angles and right-lined angles are of the ſame kind; without adding as well, that to any crooked-lined, or mixed lined angle, a right-lined angle alſo might be made equal; which can never be done; the contrary thereof being confeſſedly demonſtrated in Geometry: or if any be ſo opinionated that it is either eaſy, or feaſible, let them give a right-lined angle equal to an aniſoclitical angle, whoſe ſides concurre by way of ſection or to an ultradiametral convexo-concave angle of Contact: and trye whether the equality of the two angles be not diſproveable. In the ſame manner inequality without proportion, is aſſerted between both the recto-concave and recto-convexe angles of the ſemi-circle, *i. e.* the inner and outer angles of the ſemi-circle, and a right, or any other right-lined angle. But to return, from the concrete and compoſite nature of not right-lined angles, when compared with right-lined angles; the enigmaticalnes of the propoſition, that magnitudes of one ſort may be greater, and leſs then a given magnitude of another, but never equal, becomes moſt clear and doubtles: as infinite ſolids may be given greater, or leſſer then the heterogeneal concrete magnitude of a cube foot and a foot-line; but never can any ſolid be given thereunto equal: ſo likewiſe infinite heterogeneal concrete magnitudes conſiſting each of ſolids with the accreſcency, or annexion of a foot-line, may be given greater, or leſſer then a cube elle; but in ſuch compariſons, never can the concretes of the heterogeneal magnitudes be equal, or in proportion to any ſolitary, ſingle, one, of the heterogeneals.

In like manner it manifeſtly appears what is to be thought of that often pretended equality between the two angles of a ſemi-circle and a right right-lined angle: for by coaptation and the goings forth of the lines from the angular point it appears otherwiſe. And how will they that aſſert their equality demonſtrate it? and by what way will they admeaſure it? for interjacent and intercepted arches of Circles drawn upon the angular point as center, can in theſe angles contribute nothing to the admeaſuring of them: and coaptation makes againſt their equality: and is ſo farre from making a right right-lined angle to be the conſtant ſtandard of all angles of ſemi-circles, that it manifeſtly ſhews the angles of greater ſemi-circles to be greater and of leſs, leſſer. The objection, that unequal circles, ſemi circles, and ſegments of equal degrees, cannot be judged like and homologal figures, except all their reſpective angles be equal, as well as the ſides homologal: this may be eaſily anſwered; *Viz.* that it is like geneſis that makes like figures, which in all right-lined figures makes the anſwering angles equal, and in all figures the ſides to be homologal, and the difference of the homologal angles to be leſs then the leaſt right-lined angle.

In Circles the ſame geneſis, by the circumduction of the ſemi-diameter about the Center, which makes the figures like, and the ſides homologal, makes in unequal circles the curvatures, angles of coincidence, the angles of the ſemi circles, and of homologal ſegments, neceſſarily unequal. Or, then figures are to be judged like when in a concentricke poſition the perimeter, ſides and lines of the one proportionable to thoſe in the other, and are or may be placed parallel, or coincident, thoſe in the one, to thoſe in the other; upon which in right-lined figures, tis true, there alwayes follows equality between the anſwering angles, but not ſo in other lined figures. And becauſe equality of angles in like right-lined figures is ſo much urged; the difference of the caſe of angles in like right-lined and like crooked-lined figures may plainly appear in another remarke; *Viz.* that in like right-lined figures, the ſides of the anſwering angles may from the anſwering angular point be exactly coapted one to another; which in unequal Circles, though never ſo like figures, is moſt apparently impoſſible: that it cannot but be unreaſonable to expect as abſolute a conformity between the anſwering angles in like crooked lined figures, as there is in like right-lined figures; being ſo manifeſtly againſt both the eye and demonſtration. And therefore like ſegments of Circles, are not defined by equality of angles contained between their arches and their chords; but by the equality of the angles contained in them; the one being certain, conſtant in all, and demonſtrable; the other in moſt caſes, not only doubtful, but im impoſſible. So the objection, from the aſſerted equality of alternate angles, made by a right-line cutting parallel circumferences; is readily anſwered by denying the truth of what is preſumed in the objection, as never by any demonſtrated, or ever poſſibly demonſtrable, *Viz.* that the alternate and vertically oppoſite angles, made by a right-line cutting parallel circnmferences are eqnal. And the tyranny of forcing lines out of their natures and ſpecial properties, may appear in that very inſtance of compelling the parallelisme of curve lines to anſwer the conſectaries and idioms of the parallelisme of right-lines: in which, to omit the alleadged inſtance, as by you unproved, and for good reaſons by us to be denyed, a right-line tangent of the leſſer concentrick Circle cuts the circumference of the greater, and infinite right-lines cutting the greater, neither cut, nor touch the leſſer, which is repugnant to the nature of parallelisme in right lines. That that which is ſo much contended for, that a crooked-line and a right-line are homogeneal as to length and their general linearineſs, was, or ought never to be denied, there being all poſſibility of equality, and truly proportionable inequality between them, what kind of curvature ſoever the crooked-lines bear: but that they are homogeneal as to the poſiture of their longitude; the ſite and manner of their extenſion, hath unto me been alwayes unconceivable: whence the truths on both hands clearly follow, *Viz.* that an arch and a right-line may be equal and hold alwayes a true limited, exact proportion one to another, but the arch and its chorde never can be equal, *i. e.* there never can be equality between a right-line and a crooked-line, both poſited between the ſame two terminateing points, nor any analogy between the rectitude of the one and the curvature of the other. And the ſeeking to prove the equality of angles contain'd under homologal lines, the one curve, the other a right-line from your uſual fancy of a regular polygone of infinite angles in every circle, is too wild to be perſwaſive: for though at any mean point in curve lines, the two parts of the curve-line may be conceived ſpecially to meet as ſeveral parts and lines, and ſo to have inclination the one to the other, and ſo to conſtitute an angle; which we call the angle of curvature and coincidence, not reaſonably to be denyed by thoſe, with whom it is ſo ordinary to make ſuch ſuppoſitions, and eſpecially ſuch as can ſo uſually againſt poſſibility imagine angles in a right-line remaining a right-line: yet that angles ſhould be without ſides, and a perimeter of any figure conceived at once to be all angular points and no lineary ſides, clearly deveſtes the perimeter of the nature of a line; and to me it ſeems far from the nature of a regular figure, that hath nothing but points inſtead of lines to bound it: but which is moſt material; that a number actually infinite ſhould be ſo eaſily given, is hard to allow: and that indiviſibles, as points, ſhould be ſo adjacent one to another, one without another, without coincidence, identity and unity, is new Philoſophy, and not eaſily capable of any ingenious defenſe. Therefore that argumentation of yours, that ſuch a regular polygone of infinite whether ſides or angles is either a circle, or inſcribable in a circle, is too vain: for it can be neither, being nothing, becauſe there neither is, nor can be any ſuch thing: for if any ſuch were allowed they muſt of neceſſity have equal and infinite perimeters; which is too groſs to be admitted in it ſelf, and beſides renders the whole matter unapplyable to Circles, which are acknowledged, to be ſome leſs then others. So as all diſcourſes of a regular polygone of infinite angles, are diſcourſes not only of a non-entity, but an abſolute impoſſibility, which renders all ſuppoſitions thereof unjuſtifiable. And of the ſame fineneſs are thoſe ſayings, that the magnitude of an angle is not to be judged of from the divarication which the ſides have without the angular point, or point of concurrence, but from the

divarication which they have in the point of concurrence; as if in an indiviſible point they could have any divarication at all. But as if it were reſolved that even this ſhould be tranſcended in monſtroſity, for the juſtifying of the equality of mixed lined angles contain'd by homologal ſides in unequal Circles, by an inſtance from the coapting of unequal hexagones to the ſame line, as a common ſide in them all, divided equally by a perpendicular paſſing thorow the centers of all, a right-lined angle is ſtrangely conſtituted either of three right-lines concurring, but not in the ſame point, or of two lines without any concurrence, or elſe the inſtance muſt be void of all pertinency to the queſtion. So to all thoſe objections ſeemingly founded upon that propoſition, or poſtulate, that what is leſs then any poſitive quantity whatſoever is not any quantity at all, is juſtly anſwered; that the propoſition or poſtulate is moſt true and reaſonable, and cannot by any of ſound mind be denyed, or doubted: but no force of objection could be made out of that, if other things of a leſs veritable nature had not been taken in; as in moſt of them the fancyed poſſibility of a regular polygone of infinite angles; and frequently that a Circle is that regular polygone.

But beſides, though what is leſs then any poſitive quantity whatſoever be not any quantity at all; yet this hinders not, but quantityes may be mathematically heterogeneal and improportionable one to another: ſo every ſurface is leſs then any ſolid: and angles of Contact are not leſs then any quantity whatſoever, for there is in the leaſt of them an endles, unexhauſted diviſibility; which how it can conſiſt with a nonquantitativenes, let thoſe that have a mind to be ſe rious ſolemnely conſider. To the objection that would prove, neither ſemi-circumference to contain an angle with the right-lined tangeut of it in its extreem point, becauſe the two ſemi circumferences contain no angle at that point, but are one regularly continued line, and the circumference and right-lined tangent are lines coincident, at leaſt as to the point of Contact; manifeſt and reaſonable anſwers cannot be to ſeek out of what hath already been ſaid. For firſt what hinders the reaſonable conceiving of angularity at any point of a curve-line, where is both concurrence, inclination and diviſibility, more then the notion of diviſibility at any mean point of a right-line? And not to doubt but a curve line may be conceived reaſonably as one continued line, as well as two, or more, inclined and concurring right-lines; yet that the right-line tangent and curve-line which it toucheth, ſhould be ſaid to be coincident lines, in ſuch ſenſe as to exclude angularity;

or that any two liues can be ſo coincident in one only point, as to exclude angularity, and the inflexion of one to, or from the other, except both lye in one and the ſame right-line; hath, as elſewhere, been plainly and abundantly anſwered to. To the objection that the Area of a Circle is equal to a rect-angle under the ſemidiameter and ſemi circumference; and that therefore the ſemidiameter in a Circle is perpendicular to the circumference in a Circle and makes at the circumference four equal right-angles; is anſwered, that the whole objection is a manifeſt paralogiſme. For it is not denyed but in the right-lined rect-angle under the ſemidiameter, and a right-line equal to the ſemi circumference, is preſumed, and by the definition of a rect-angle inferred, that the angle under thoſe two right-lines is a right right-lined angle: beſides it is not denyed, but the diameter falls perpendicularly in the circle upon the circumference: and that the four angles made by the falling of the ſemidiameter upon the circumference differ from one another leſs than the leaſt right-lined angle: however that cannot force the falling of the ſemidiameter perpendicularly upon the circumference into the properties of perpendicularneſs between right lines, which ſtill divides the ſpace at the angular point into four angles always, every way alike, and equal; which in right lines perpendiculars upon curve lines in the ſame plane, is impoſsible to be, and therefore impoſsible ever to be demonſtrated.

It will not be unuſeful here to enquire wherein the likeneſs and unlikeneſs of angles doth conſiſt, and whether there be any ſuch thing as likeneſs and unlikeneſs in angles, or whether the likeneſs, or unlikeneſs of figures be only in the ſimilitude, or diſſimilitude of the ſides. And that by a circumſpect conſideration of the nature of mathematical ſimilitude in other caſes, we may be the better guided into the true and moſt rational notion of ſimilitude in angles, let us remember what hath already been judged in this point, and what is herein confeſſed on all hands. Firſt, in right lined figures, thoſe figures are judged like, whoſe anſwering angles are equal, and the anſwering ſides and other lines proportionable; and if they be equal, they may be coapted, homologal ſide to homologal ſide, and anſwering angle to anſwering angle; or whether they be equal, or unequal, all the ſides and other anſwering lines of the one may be ſet, as from the ſame center, each at parallelisme, or coincidency with the anſwering ſides, or lines of the other, ſo as in like right lined figures is proportionableneſs in the anſwering ſides, equality of the anſwering angles, coaptability of all the anſwering ſides into either coincidence, or equidiſtance, and a proporportionate diſtance of the anſwering angles, each from the other. But now the ſimilitude of figuration which is in circles founded upon the like geneſis of all circles is in the equidiſtance, or coincidence of their circumferences, when the center of the one is coapted to the center of the other; and that equal angles from the center intercept proportionable parts of the circumferences, and that proportionable parts of the circumferences are connected by proportionable chords, and contain and ſuſtain equal right lined angles. And the like ſpeculations might be purſued in other figures both plane and ſolid. In a rational application of which to the diſquiſition of angles, it may be firſt enquired, whether there be any ſuch thing as ſimilitude & diſſimilitude to be own'd or obſerv'd among angles; and if ſo, how that ſimilitude is to be underſtood; and whether it be inconſiſtent with inequality in the anſwering angles. To clear all which we muſt know, that in unequal but like right lined figures, the homologal angles are always equal, being contained in both figures under right lines: but in unequal, and like mixed lined and crooked lined figures, the homologal mixed lined, or crooked lined angles, neither are, nor can be equal; only their difference is ever leſs than the leaſt right lined angle; and their ſimilitude hath never rationally yet by any been queſtioned but with good reaſon according to the following gloſs is to be juſtified. The more clearly to demonſtrate all which in Fig. 21. upon the common center A. draw two unequal Circles, *Viz*. HEG. the leſſer, and BCD. the greater. Then from any point B. in the greater circle BCD. draw the right line BEAFK. thorough the common center A. cutting the circumference of the leſſer circle HEG. in the point E. then take AF. equal to BE. and upon the center F. and ſemidiameter FE. draw another circle ECKD. equal to the greater circle CBD. Here on all hands is agreed, that the leſſer circle HEG. and the greater circle CBD. are like figures, and that therefore the two mixed lined recto-concave angles ABD. and AEG. are like angles. And by the conſtruction it is apparent that the two recto-concave angles ABD. and AED. are equal: and that the two recto-concave angles AEG. and AED. are unequal: and that the recto-concave angle AED. is greater than the recto-concave angle *AEG*. and in all like caſes it is always ſo; however the difference of the two angles muſt neceſſarily be leſs than any right lined angle, becauſe all ſuch citradiametral concavo-convexe angles of contact, as GED. are always leſs than any right lined angle; as is conſequent to what hath been demonſtrated in Geometry, which was to be ſhewn. Whence we may clearly obſerve that ſimilitude of Figures lies chiefly in the proportionality and like poſiture of homologal ſides, in reſpect of paralleliſme and coincidence, without impoſing any other neceſſity for the equality of anſwering angles, then as it may conſiſt with the proportionating and like poſiting of the homologal ſides and lines. And ſuch inequality of the anſwering angles; as is requiſite to the proportionating and alike poſiting of the homologal ſides and lines in like and unequal mixed, or crooked lined figures, is ſo far from being inconſiſtent with their figurative ſimilitude, that they cannot without it, under inequality keep ſimilitude in their figuration. And though the inequality, which is between angle and angle be leſs then that which is, or may be, between the Homologal ſides and lines; yet the inequality of the angles is more different, being an inequality without proportion, whereas the inequality of the homologal ſides and lines is ever according to proportion. Upon the whole, it is not equality that generally makes angles to be like, for a right lined, and an Iſoclitical concavo Convexe may be equal angles, but never can be like, nor were ever ſuſpected to be ſo: but that which makes angles to be like is rather their being contained under homologal ſides, poſited ſo as to conſtruct a like & homologal figuration. And this whole matter depends upon what I before hinted, *Viz.* the figuration of lines and angles; *Sc.* rectitude, being one ſingle, ſimple, figuration of lines incapable of any variety, like angles under right lines are always equal, and never can be unequal: but curvature, being infinitely variable, thoſe crooked lines are ſaid to be like, *i. e.* homologal, whoſe conſtruction is like, ſo as in like figures upon a common center, to ſet homologal ſides and lines proportionably equidiſtant, or coincident; as circumferences of like though unequal circles, ellipſes, &c. And ſo under a thouſand inequalities ſuch mixed and crooked lined angles may be like: as in Fig. 21. the recto-concave angles A E G. and A E D. being unequal, are both like to the angle *A* B D. and ſo is every angle how different ſoever, if contained under a diameter and a circumference. *A*nd indeed the figuration of angles, being incompleat, and the length of their ſides undetermined, neither paralleliſme, nor coincidency, nor proportionality, nor homologal poſiture, can, when they are unequal, be conceived in their ſides, without ſpecial relation to ſome compleat Figure and its Center: ſo the recto-concave angles *A* E G. and *A* B D. in Fig. 21. are like, as conceived to be each contained reſpectively under a diameter and a circumference, and ſo upon a common center poſitable into paralleliſme, coincidence and proportionableneſs, and all poſſible likeneſs and homolologalneſs of figuration. In right-lined angles, where homologal and like angles are always equal; for the ſame reaſon every angle equal to a right-lined angle, is not preſently a like angle: but a thouſand equal angles, are all, ever, and to all purpoſes, unlike; as two equal iſoclitical right lined and crooked lined angles, becauſe they can never be coapted to be anſwering angles, in like Figures, or to ſet their containing ſides homologally, and in parallelіſme, or coincidence. That equality of anſwering angles is not ſo of the eſſence of like Figures, as proportionality of ſides, and anſwering lines, with their paralleliſme, or coincidency: only from the propriety of like plane Figures, follows an equality in all like right-lined angles; and in like curve lined Figures, that their inequality is ever leſs than the leaſt right-lined angle. Hence therefore appears that from the ſimilitude which is in unequal circles, the equality between angles of ſemicircles & right right-lined angles is not effectually proved. And notwithſtanding any thing in thoſe arguments tendred aud proved, every recto-concave angle contained under a concave arch of a circle and a right-line, which is perpendicular to the right line tangent of the arch at the angular point, is greater than any right-lined acute angle, and leſs than a right right-lined angle: and the recto-convexe angle contained under the convexe arch and the right lined tangent is leſs than any right lined angle whatſoever: and the other recto-convexe angle contained under the convexe arch and the right line, which is perpendicular to the right line tangent at the angular point, is greater than a right right-lined angle, and leſs than any obtuſe right-lined angle whatſoever. *A*nd whereas you object that if, as in Fig. 12. the right line K G *A*. be the diameter of the circle K *A* D. and *A*B the right line tangent, then *K A B* is a right right lined angle: and the recto-convexe angle of contact *D A B* is no part of the right right lined angle *K A B*. that therefore the angle of the ſemicircle *K A D* is ſtill equal to a right right lined angle, becauſe what is taken out of it was no part of it. I anſwer, the recto-convexe angle of contact *D A B* is indeed no proportionable part of the right right-lined angle *K A B*, but yet it is truly a part, though improportionable, and ſo mathematically heterogeneal: for if it had been no part at all, and nothing, then the angle of the ſemicircle *KAD*. (nothing being taken out of the right right-lined angle *K A B*, but the recto-convexe angle of contact D *A* B, which is ſaid by you to be nothing, and no angle) it ſhould ſtill remain a right right-lined angle; which is not by any aſſerted, the contrary being ſo manifeſt: beſides that the ſeparability of the recto-convexe angle D *A* B from the recto-concave angle *KAD* makes clear and certain, the truth of its being a part of the right-lined angle KAB.

And likewiſe from what hath been before declared in our opening the nature of a plane angle may clearly appear, that we are not to underſtand that a plane angle is meerely the angular point, or meerely in the angular point, as contradiſtinguiſhed from the containing ſides, though it there terminate, or thence have its riſe; but angles are in the habitude of the concurring, containing and inclined ſides: *Viz.* the habitude which they hold each to other all along their tendency unto the angular point, or their riſe from thence, if we would have the full notion, inclination and figuration of an angle. For there is often a great inequality and vaſt imparity between the inclination, ſometimes of one part of the containing ſide to the other containing ſide, and the inclination thereunto, of other parts of the ſame firſt containing ſide; as may appear in all mixed lined, mixed crooked-lined and all other aniſoclitical angles. And the nature of an angle conſiſting in inclination as well as in concurrence, though concurrence may be and is in a point, and inclination at a point, yet inclination muſt be in the lines and of the lines, and cannot be in a point ſeparately. And methinks the nature of an angle, and its inclination, is

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ſcarcely ſo fully held forth, when the inclination of the two lineary ſides containing it, as if the ſides were not therein concerned, is ordered to be obſerved only in the angular point, and not out of it: becauſe, as you ſay, though you urge it to the contrary, many times out of the angular point in the containing ſides, no two points can be ſhewen in the one ſide, where it hath the ſame inclination unto the other. Certainly in an indiviſible, ſuch as is the angular point, if abſtractly conſidered, it were vain to expect and impoſſible to obſerve any inclination: and no doubt as the magnitudes inclined are without the angular point, ſo is alſo the inclination: though as they terminate in the angular point, ſo doth the inclination. So the angle of a ſemi circle is not the common terme of the diameter and the ſemi circumference, excluding the diameter and the ſemi-cir-cumference for then in an abſtracted point it ſhould be poſſible to obſerve an inclination; and a point being indiviſible ſhould be inclined unto its ſelf; which is not convenient to aſſert: but rather the angle of a ſemi circle is the inclination of the ſemi circumference to the diameter, terminated in the angular point, which is common to both. And whereas it is ſaid that out of the angular point, no two points can be ſhewen in the diameter, at which the diameter is equally inclined to the ſemi circumference, nor in the ſemi circumference where it is equally inclined to the diameter; all is allowed and averred as gloſſeably true, and this is that which makes the great difference between aniſoclitical and iſoclitical angles, and renders it ſo impoſſible to give an aniſoclitical angle equal to an iſoclitical angle. For in iſoclitical angles the inclination of the ſides, the one unto the other, is at all points the ſame without any variation; as every where appears by the interjected arches of circles drawn upon the angular point as Center: but in aniſoclitical angles, at every ſeveral point, the one containing ſide hath ſeveral and different inclinations to the other containing ſide: which is the cauſe that iſoclitical angles may poſſibly and eaſily be given, ſometimes greater, and ſometimes leſſer then aniſoclitical angles, but never equal: becauſe in the one the inclination of the containing ſides ſtill varieth, in the other not at all. And if the whole nature of an angle lye in the angular point, without extending the habitude of it farther into the production and figuration of the containing ſides; it will not only be neceſſary for us to yield unto you that recto-convexe angles of contact are not quantitative; but beſides both you and we, contrary to what we have alwayes hitherto judged, ſhall be conſtrained to acknowledge that there is no quantitativenes neither in crooked-lined, nor right-lined, nor any other angles whatſoever, whether ſuperficial, in one, or ſeveral plaines, or ſolid. And can any thing be more horrid then to ſay, the quantity of angles is not to be meaſured by the divarication of the ſides at the angular point, but by their divarication in the angular point, where they have none at all? But yet though it is thus evident, that the inclination of the ſides at the angular point, may and frequently is much leſs or greater then the inclination of the ſame ſides at other points; which, as is above hinted, is not to be left out in the full, genuine and clear conſideration of the nature of angles, their kinds, figurations and quantities: however the inclination of the ſides at the angular point, is that which is moſt uſually enquired after, and moſt uſeful to be ſearched and obſerved in Geometry; for the diſcoveries which are from thence made of lines, how they fall coincidently, or within, or without others. To the objection that in fig. 12. the right-lined tangent AB and the arch AL make all one and the ſame equal inclination to the right-line ſecant AC in the common angular point A; and that therefore the right-lined angle BAC under the right-lined ſecant AC and the right-line tangent AB is equal to the mixed-lined recto-concave angle EAL under the right-line ſecant AC and the arch AL; I anſwer as before, inclination is not in the angular point abſtractly conſidered without regard to the ſides paſſing out of it, but inclination is the relative ſituation which the concurring ſides have at the angular point: at leaſt that is their inclination there: for a point to a line can have no inclination: it may have diſtance from the line, but cannot be inclined unto it, becauſe of its indiviſibility. *A*nd having already ſhewn the inclination of ſide to ſide to be of the eſſence, notion and nature of an angle; a little may be reply enough to all thoſe hyperſceptical objections which are founded upon the imagination of an angle in a right-line, or any inclination, or angularity imagined between a right line and a point, eſpecially the point being in the right-line. And equality, or inequality of angles is not, nor can be judged of by the ſo abſtractly conſidered angular point; in which a thouſand ſeveral ſides of ſeveral and unequal angles may meet indifferently; but the judgment of the magnitude and equality and inequality of angles is from the ſides, and the order of the divarications, in which they paſſe, eſpecially firſt of all, from the angular point. Beſides how ſtrangely is it taken for granted, without proving, that the right-line tangent AB and the arch *A*L are equally in the angular point *A* inclined unto the right-line ſecant *A*C? If that could be once proved, the concern of it would turn the ſcales of the controverſy: but demonſtration is ſo clear to the contrary, that as without proof it is not fit to be admitted, ſo for the proof of it, I know nothing can be produced beſides an utter deſpair of ever making it out.

For if the congruency of the ſides terminatively in the angular point, were ſufficient to conſtitute equality in angles, it appears not how any angles meeting in the ſame or different angular points could be unequal: every point by reaſon of its indiviſibility being incapable of inequality, as well as inclination.

And if all ſuch angles ſo conſtituted by the falling within, or without of the ſides, ſhall be doubted and queſtioned whether they be true and quantitative angles, and whether the addition, or ſubduction of them be able to diverſify other angles and their quantities; all the pains of the Geometricians to prove the intracadency and extracadency of the angular ſides from the ſame augular point, were vain and to no purpoſe; the angles remaining altogether the ſame and equal, whether ſuch angles of contact be added to them, or taken from them. But yet though the true, full and genuine nature of an angle conſiſt in the mutual habitude and inclination of the containing and concurring ſides, it is not ever neceſſary to conſider it with ſuch a largenes in Geometry. The inclination which the ſides bear mutually each to other at the angular point is out of doubt that which is of moſt conſtant neceſſity, higheſt concern and uſefulnes in all angles to be obſerved. It is true in iſoclitical angles, the inclination of the two containing ſides being every where the ſame and equal, it is indifferently by Geometricians taken by a circle whoſe center is in the angular point, of what diameter ſoever thereunto applyable, and at what ſame diſtance ſoever from the angle, or at what ſame longitude ſoever from thence in the ſides the points be, at which their inclination is obſerved by intercepted arches: and the ſame two points, terminating the intercepted arches at which their mutual inclination is obſerved, conſtantly offer themſelves together, whether you take points at equal diſtance from the angular point, or intercepting in the ſides equal longitudes between them and the ſame angular point. But in aniſoclitical angles; the inclination of the aniſoclitical ſides varying ſtill in the continuity of their production, if we uſe the former method of meaſuring the inclination of the ſides by intercepted arches of a circle drawn upon the angular point as center; in them therefore Geometricians concern themſelves little further then to obſerve the mutual inclination of the ſides at the very point of their angle: and not at any other points in the aniſoclitical ſides, ſave only the point of their concurrence; becauſe of the couſtant variation of their inclination, both in reſpect of ſuch, and every other method and way of meaſuring, according to the continuity of their production. *And* by arches of circles drawn upon the angular point as center, it is impoſſible to meaſure the inclination of the aniſoclitical lines at the point of their concurrence; the only way therefore which remains unto Geometricians to meaſure ſuch aniſoclitical angles, *i. e*, the inclination of their ſides at the point of their concurrence, is by obſerving the lines in what order they depart from the angular point: *Viz.* which line falls within, which without, and which is coincident with that unto which it is compared. So in fig. 12. if AHF and ADK be equal circles touching in the point A: and G the center, and *A*GK the diameter of the circle *ADK*:and *AEL* the arch of a greater circle touching both the former circles in the ſame point A and with its concave ſide at *A* reſpecting the center G, and with its convexe ſide at *A* reſpecting the circle *HAF*: alſo if *AB* be a right-line tangent touching all the three former circles in the point *A*, aud the right-line *ADEC* cutt the circle *AD*K in the point [illegible], and the circle *AEL* in the point *E*: here the Geometrician demonſtrates the angle K*AB* to be a right right lined angle, and that the arch *AF*H paſſeth out of the angular point *A* without and beyond the right-line tangent *AB*, and without the right right-lined angle K*AB*: and that the arch *AEL* paſſeth from the angular point *A* within, or on this ſide of the right-line tangent *AB*, and within the right right-lined angle KAB, but without the arch ADK, and without the angle of the ſemi circle KAD: and that the right-line *ADEC* paſſeth from the angular point *A* within the arch *AD*K, and within the angle of the ſemi circle K*A*D. *A*nd this is all which is intended in Geometry when recto-convexe and ſuch like angles of contact are ſaid to be leſs then the leaſt right-lined angle: not that there is any proportion between any angles of contact and right-lined angles; for there is none: but that the ſides of any ſuch angle of contact are coaptable, both of them within any the leaſt right-lined angle. In like manner when one angle of contact is ſaid to be greater or leſſer then another; it is not the intendment of the Geometrician to aſſert any proportionablenes between them, but only to ſet down in what order the inclined lines paſs, each in reſpect of the reſt, from the angular point, or point of concurrence: and how each is conſequently more, or leſs inclined in that reſpect, to any one of the reſt, though by the whole kind and without proportion, and without a common way of meaſuring their quantities in ſome indefinite quantity, according to the kind of the meaſuring, indefinite quantity: the ſemblablenes of ſaying, one angle of contact is greater, or leſs then another being only in this, that when one ſide is coapted, the other falls within, or without; or when neither ſides can poſſibly be coapted, both fall within, or both fall without, ſo as the one is really greater, or leſs then the other, not in proportion, but by the whole kind and heterogeneally, as a two foot right-line is leſſer then a four foot ſquare, and a two foot right-line cannot be contained in an inch ſquare, though in an inch ſquare may be drawn a line ten thouſand times longer, only out of rectitude. *A*nd to prove ſuch an heterogeneal inequality, whether of the greater, or of the leſs between angles, is that which is frequently ſufficient for many Geometrical purpoſes.

Only two things now remain of all your objections and ſcrupulous quæries. *Viz.*

Firſt, why in ſome quantities, between the homogeneals, to a given quantity you cannot give another in what proportion you pleaſe greater or leſs, The anſwer is clear out of the foregoing diſcourſe: becauſe ſome quantities have but a limited extenſibility: and other quantities have but a limited diviſibility. *A*ll proportions at pleaſure with reſpect to any given quantity, are only aſſigneable in thoſe homogeneals whoſe quantityes have both an unlimited extenſibility and an unlimited diviſibility; either of which failing, though they cannot be, ſave in a limited proportion one to another, and to every given homogeneal, yet they cannot be in any proportion, at pleaſure, to any given homogeneal:

And 2^{ly}. for your other Quæry; whether this controverſy about the recto-convexe angle of contact concern abſtract Mathematicks, whoſe glory uſed to be that it was devoid of Controverſy, locking forth and keeping out all unpeaceful bickerings with the diamond-key of doubtleſs demonſtration; or whether it concern concrete Mathematicks, which for its concretion unto matter cannot ſo well free its ſelf from the intanglements of doubts and diſputes? and if it concern the more abſtract part of the Mathematicks; what was the firſt ſlip that drew ſo unworthy a diſparagement upon that moſt noble piece of learning? In the preceding diſcourſe you may obſerve, I have been free and clear in my judgement concerning this; that it is a controverſy of the purer and more noble part of the Mathematicks: and therefore of the higher concern, that it be brought to a fair deciſion and irrefragable diremption: and that the firſt ſpark from whence all this heat afterward aroſe, was at firſt ſtruck out of the dubiouſnes and æquivocation of the word homogeneity, mentioned in certain Mathematical definitions and Poſtulates; but of it ſelf no where expreſſely defined in the Mathematicks; except we ſhall take one of the Puſtulates, for its definition caſt into the form of a Poſtulate; or as cryptically implying that definition and the poſtulate to be thereupon immediately formed: eſpecially when they compared homogeneity in a miſtaken ſenſe, with ſome mathematical concluſions, which they had obſerved to be fairly demonſtrated.

Pardon this my zeal on the behalf of the old learned Romane profeſſour. And it theſe papers ſeem long, remember yours were not ſhort; and the queſtion hath long troubled the world: and in matters that are new, and ſcarce yet well underſtood, to inculcate once and again the ſame things, is not only juſtify'd as allowable, but judged expedient and in ſuch caſes neceſſary. If you pleaſe, let the whole paſſe for a fuſe Comment upon this; that in recto-convexe angles of contact is the inclination required for an angle by the definition of an angle; and therefore they are angles and their ſides not coincident: and they are angular parts of acknowledged angles ſeparable from the remaining angular parts; and there is an infinite diviſibility in them, ſo as they muſt be quantitative: and yet it is demonſtrated that they are improportionable to all right-lined angles; therefore they are heterogeneal: which in-

inference is the more neceſſary, when upon examination we find nothing ſo convenably to ſet forth unto us the nature of mathematical homogeneity, as the homometricalnes and the thence ariſing proportionablenes of the magnitudes, and of mathematical heterogeneity as their heterometricalnes and improportionablenes.